MSUCP-6

MICHIGAN STATE UNIVERSITY

CYCLOTRON PROJECT*

Survey of Variations in Orbit Characteristics Arising from Azimuthal Shifting of Cos Θ Field Component in a Resonant Extraction System for a 3 Sector Cyclotron

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I. INTRODUCTION

If a field component with cos Θ azimuthal dependence is added to the magnetic field of a 3 sector medium energy cyclotron, approximate solution of the equations of motion of the radial betatron oscillation[1] leads to phase space

1. M. M. Gordon, MSUCP report, in preparation. A somewhat less general equation has been extensively treated by L. J. Laslett MURA 452, 459, 461, 463, 490 and 515, unpublished.

diagrams of the type shown in Fig. 1. Sketch (a) of the figure depicts the features of the phase diagram in the absence of a cos Θ field component and sketches (b) and (c) depict the features for two special azimuthal locations of the cos Θ component, the two locations resulting in motions of a pure character which we designate "one corner opening" and "two corner opening" respectively following the obvious features of the phase diagrams. For other azimuthal locations of the cos Θ component, an appropriate mixture of one corner and two corner openings is predicted.

In the design of resonant extraction systems it is essential that the family of trajectories representing the beam spot stay well clear of the unstable fixed points if severe distortion of the beam is to be avoided. This condition is most easily satisfied if the cos Θ component is azimuthally positioned to yield one or the other of the two pure states. The studies described in this report were undertaken to establish, for the magnetic field of the proposed MSU cyclotron, the specific azimuthal locations of the cos Θ component which result in the pure states. Semi-empirical relationships are

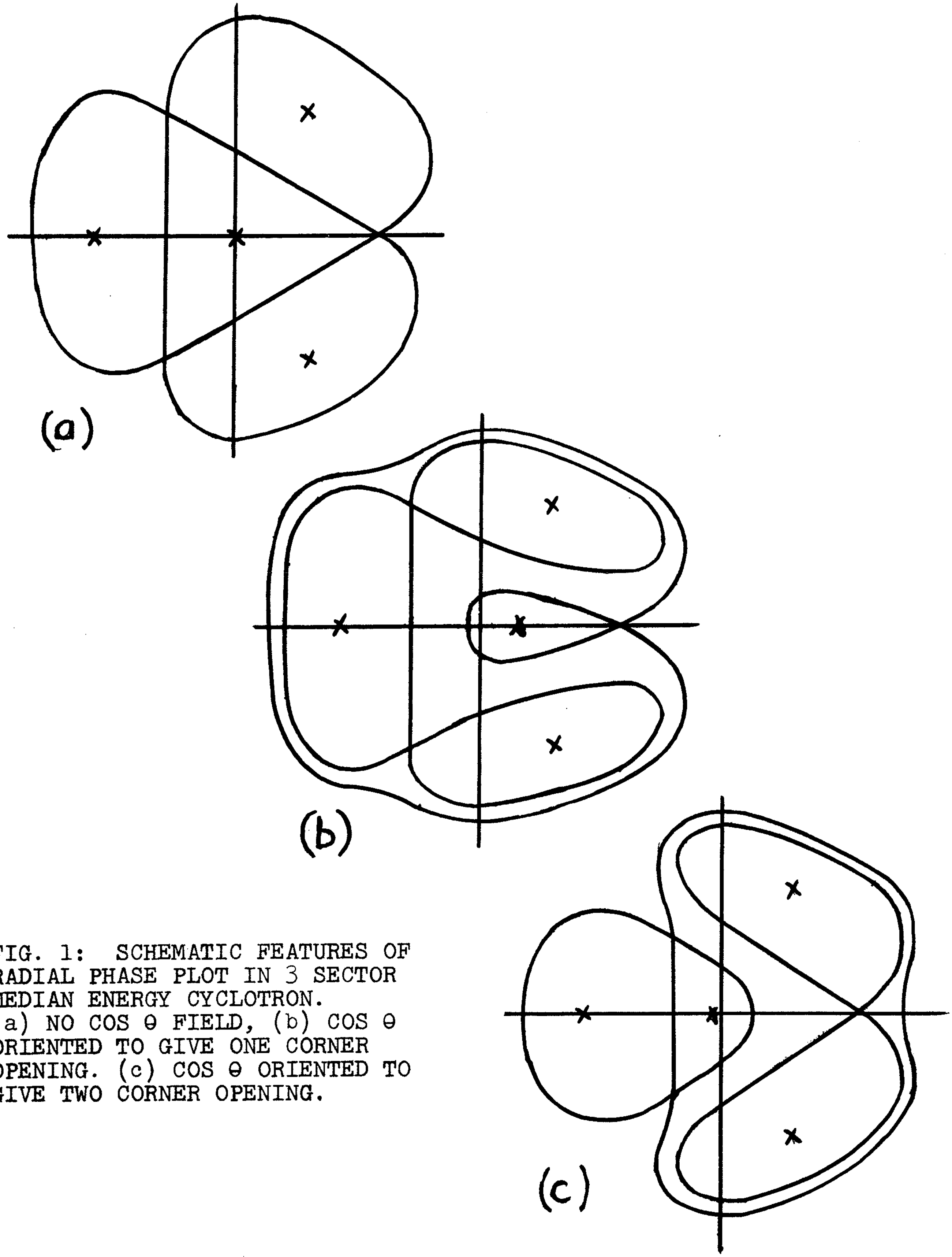

FIG. 1: SCHEMATIC FEATURES OF RADIAL PHASE PLOT IN 3 SECTOR MEDIAN ENERGY CYCLOTRON.
(a) NO COS Θ FIELD, (b) COS Θ ORIENTED TO GIVE ONE CORNER OPENING. (c) COS Θ ORIENTED TO GIVE TWO CORNER OPENING.

developed relating the preferred azimuths to the azimuths at which various orbit symmetries occur: these relationships will quite probably be valid in magnetic fields differing considerably from the one considered here. Several mixed states have also been explored in order to better understand their properties and the results are included.

II. CALCULATION PROCEDURES

Orbit computations have been made using the Fixed Point Code[2] and the General Orbit Code[3].

2. T. I. Arnette, M. M. Gordon, and H. G. Blosser, MSUCP report in preparation.
3. M. M. Gordon and H. G. Blosser, Mistic General Orbit Code MSUCP-7 (1960) unpublished.

for the MSU computer the MISTIC. The Fixed Point Code locates closed orbits by means of a highly effective linear transfer matrix procedure, the General Orbit Code tracks arbitrary orbits as desired either with or without acceleration effects. For both routines the magnetic field is described by tables of Fourier coefficients as functions of radius; each uses equations of motion which are exact in the median plane.

To delineate the features of a particular phase diagram, the fixed point routine is first employed to locate the various closed orbits (occasionally for expediency only the unstable fixed point orbits are located) after which the general orbit code is employed to trace forward and backward in time orbits with initial conditions displaced slightly from the unstable fixed points. These orbits to good approximation represent the boundaries of the various topological regions of the plot which are the principal feature of interest.

The magnetic field employed in these studies is given in Table I. The field is derived from a model magnet study known as Run 26[4] with modifications to ⟨B⟩ to yield

4. H. G. Blosser and D. A. Flanigan, Run 26 Field Information, MSUCP-3 (1960) unpublished.

approximate isochronism out to the 29th entry in the radial table and with the flutter field modified by a small amount to smooth out effects of measurement errors. In addition Fourier components of argument greater than 9 θ have been dropped since these components are sufficiently small to have a negligible effect on the particle motion. The radial spacing of the table entrys is interpreted as increased by the factor 64/8.75 corresponding to the ratio of pole diameters of the proposed MSU cyclotron and the Run 26 magnet.

The amplitude of the cos θ field component employed in the studies is given in Table II. The radial profile corresponds to the shape expected in the MSU cyclotron where a set of coils will be relied on to produce the cos θ field. At one cos θ orientation an additional run was made using a cos θ uniform with radius in order to establish the degree of sensitivity of the phenomena to variations in the radial profile.

r MODEL INCHES	B_o GAUSS	H_3 GAUSS	G_3 GAUSS	H_6 GAUSS	G_6 GAUSS	H_9 GAUSS	G_9 GAUSS
0.1	+13623	+00025	-00004	-00019	-00019	+00008	-00030
0.2	+13624	+00060	-00100	+00018	+00027	+00000	+00005
0.3	+13625	+00187	-00302	+00023	+00045	-00006	+00009
0.4	+13624	+00378	-00570	+00010	+00043	-00009	-00003
0.5	+13620	+00612	-00872	-00009	+00026	-00007	-00023
0.6	+13614	+00875	-01185	-00027	+00002	+00000	-00042
0.7	+13607	+01159	-01488	-00041	-00025	+00012	-00053
0.8	+13600	+01457	-01768	-00047	-00052	+00028	-00053
0.9	+13593	+01766	-02016	-00047	-00074	+00046	-00040
1.0	+13587	+02081	-02227	-00042	-00089	+00065	-00013
1.1	+13583	+02395	-02399	-00035	-00095	+00084	+00026
1.2	+13579	+02706	-02531	-00028	-00091	+00100	+00077
1.3	+13578	+03007	-02624	-00027	-00077	+00111	+00136
1.4	+13578	+03296	-02682	-00032	-00052	+00116	+00201
1.5	+13580	+03570	-02708	-00047	-00017	+00113	+00268
1.6	+13587	+03825	-02703	-00073	+00028	+00102	+00333
1.7	+13601	+04062	-02672	-00111	+00080	+00081	+00395
1.8	+13617	+04282	-02618	-00161	+00139	+00052	+00450
1.9	+13634	+04479	-02544	-00222	+00202	+00015	+00496
2.0	+13651	+04660	-02452	-00295	+00266	-00029	+00531
2.1	+13669	+04823	-02347	-00379	+00330	-00078	+00556
2.2	+13690	+04968	-02232	-00471	+00390	-00130	+00570
2.3	+13714	+05097	-02107	-00571	+00445	-00183	+00572
2.4	+13737	+05207	-01977	-00678	+00491	-00233	+00564
2.5	+13761	+05300	-01842	-00790	+00527	-00278	+00546
2.6	+13792	+05376	-01704	-00905	+00550	-00316	+00520
2.7	+13827	+05434	-01564	-01022	+00558	-00345	+00486
2.8	+13861	+05476	-01422	-01140	+00551	-00363	+00447
2.9	+13890	+05505	-01278	-01258	+00528	-00368	+00404
3.0	+13907	+05523	-01133	-01374	+00488	-00359	+00358
3.1	+13917	+05534	-00987	-01486	+00432	-00337	+00313
3.2	+13918	+05541	-00840	-01593	+00362	-00301	+00268
3.3	+13911	+05545	-00693	-01693	+00278	-00253	+00227
3.4	+13895	+05545	-00547	-01783	+00186	-00197	+00191
3.5	+13864	+05538	-00404	-01859	+00087	-00133	+00160
3.6	+13805	+05517	-00265	-01919	-00013	-00068	+00137
3.7	+13709	+05471	-00135	-01959	-00110	-00004	+00121
3.8	+13564	+05386	-00016	-01973	-00199	+00054	+00114
3.9	+13352	+05249	+00085	-01958	-00276	+00104	+00114
4.0	+13046	+05046	+00166	-01909	-00337	+00141	+00120
4.1	+12608	+04765	+00223	-01824	-00379	+00164	+00132
4.2	+12001	+04396	+00252	-01701	-00399	+00172	+00147
4.3	+11231	+03935	+00255	-01542	-00396	+00166	+00161
4.4	+10343	+03390	+00234	-01352	-00371	+00148	+00171
4.5	+09408	+02785	+00194	-01139	-00326	+00123	+00171
4.6	+08497	+02172	+00146	-00917	-00265	+00098	+00155
4.7	+07661	+01640	+00102	-00704	-00195	+00083	+00115
4.8	+06930	+01317	+00083	-00525	-00123	+00090	+00044

TABLE 1: TABLES GIVING FIELD COEFFICIENTS VS. RADIUS. IN THE CALCULATIONS RADIAL SPACING IS INTERPRETED AS INCREASED BY FACTOR 64/8.75 RESULTING IN RADIAL INCRENENT IN CYCLOTRON UNITS OF 0.0080924.

$$B(r, \theta) = Bo(r) + \sum_{j=1}^{3} (H_{3j} \cos 3j\theta + G_{3j} \sin 3j\theta)$$

r MODEL INCHES	B_1 GAUSS
0.1	00000
0.2	00000
0.3	00000
0.4	00000
0.5	00000
0.6	00000
0.7	00000
0.8	00000
0.9	00000
1.0	00000
1.1	00000
1.2	00000
1.3	00000
1.4	00000
1.5	00000
1.6	00000
1.7	00000
1.8	00000
1.9	00000
2.0	00000
2.1	00001
2.2	00002
2.3	00004
2.4	00009
2.5	00014
2.6	00022
2.7	00034
2.8	00053
2.9	00080
3.0	00109
3.1	00131
3.2	00139
3.3	00131
3.4	00109
3.5	00080
3.6	00053
3.7	00034
3.8	00022
3.9	00014
4.0	00009
4.1	00004
4.2	00002
4.3	00001
4.4	00000
4.5	00000
4.6	00000
4.7	00000
4.8	00000

TABLE 2: AMPLITUDE OF COS θ FIELD COMPONENT VS. RADIUS.

III. RESULTS

Tracing of fixed points as a function of energy showed that for the field here considered, and with a 1% cos θ component, the normal stable region of the phase plot vanished typically in the vicinity of 31 Mev. It appeared desirable for this region to show up in the survey of cos θ azimuths and so 30 Mev was selected as the energy at which to make the survey. Fig. 2 is a phase plot at 30 Mev in the absence of a cos θ field component. Figures 3 through 8 are phase plots with cos θ component of amplitude and radial profile as given in Table II and with the azimuthal maximum of cos θ located at the series of angles -33.9°, -26.1°, -2.8°, $+17.3^\circ$, $+33.9^\circ$ and $+45.0^\circ$ with respect to the main field, the total field being given by the expression

$$B(r,\theta) = B_0(r) + \sum_{j=1}^{3}\left(H_{3j}\cos 3j\theta + G_{3j}\cos 3j\theta\right) + B_1(r)\cos(\theta - \phi). \qquad (1)$$

Fig. 7 is quite nearly a pure one corner opening, Fig. 3 is quite nearly a pure two corner opening while the other four figures display various mixtures of one and two corner openings. From the figures it is inferred that the pure one corner state occurs at $\phi = 32^\circ$, and the pure two corner state at $\phi = -37^\circ$. Fig. 9 is analagous to Fig. 7 except that B_1 has been changed to 125 gauss amplitude uniform with radius. In all of the figures 2 through 9, the curves are arbitrary lines drawn to connect points of a given trajectory. At several

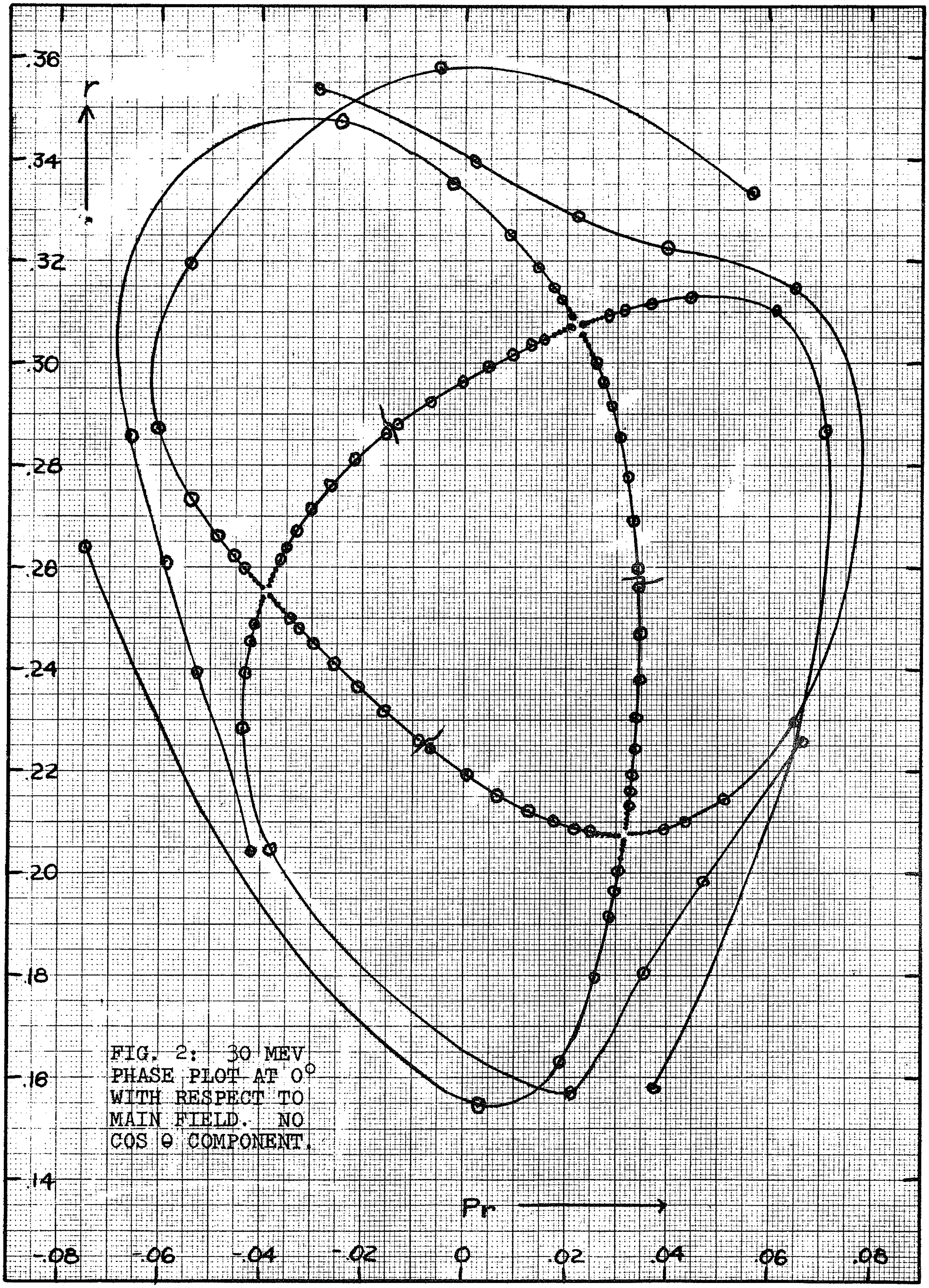

FIG. 2: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. NO COS Θ COMPONENT.

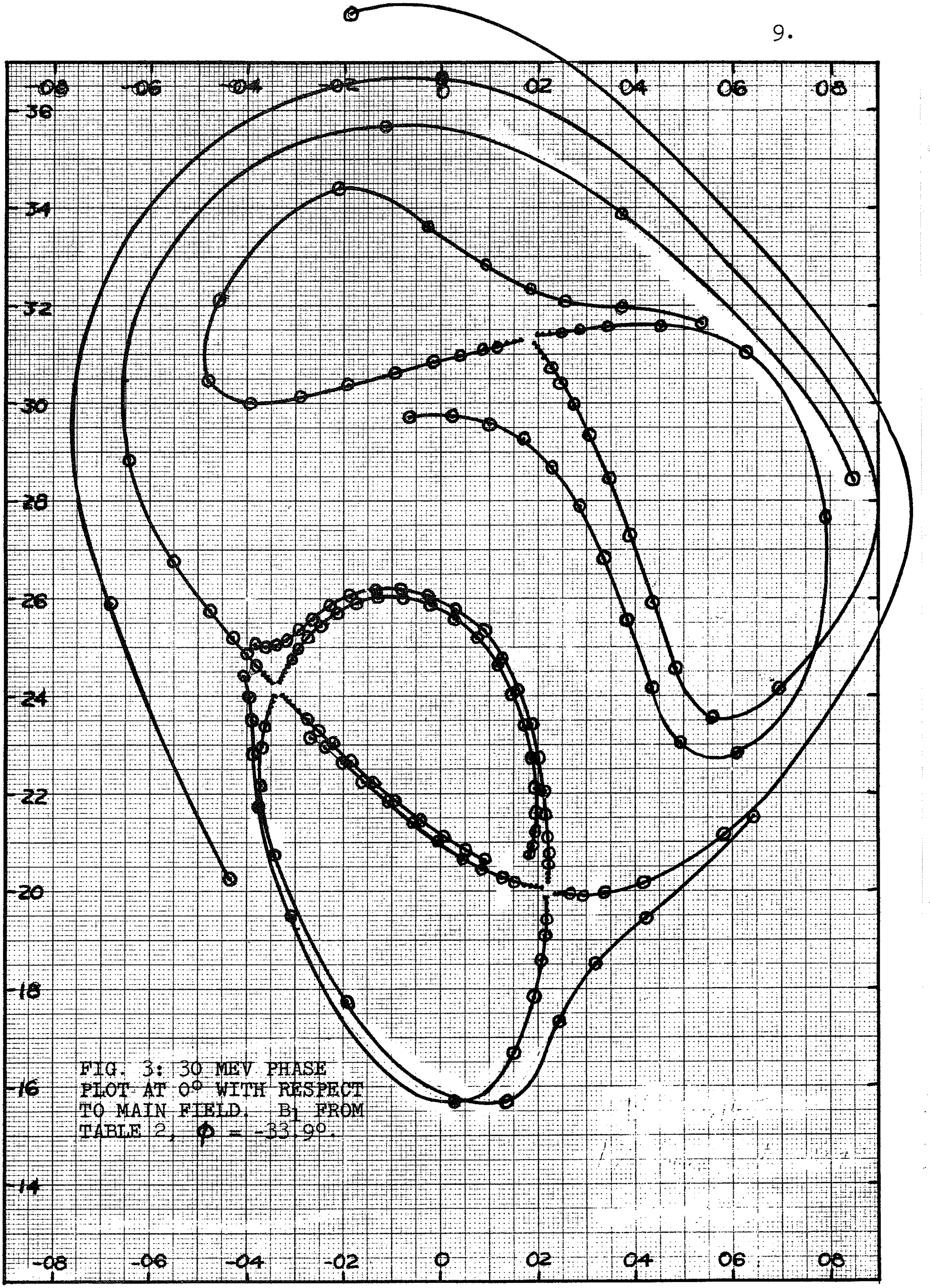

FIG. 3: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. B_1 FROM TABLE 2, $\phi = -33.9°$.

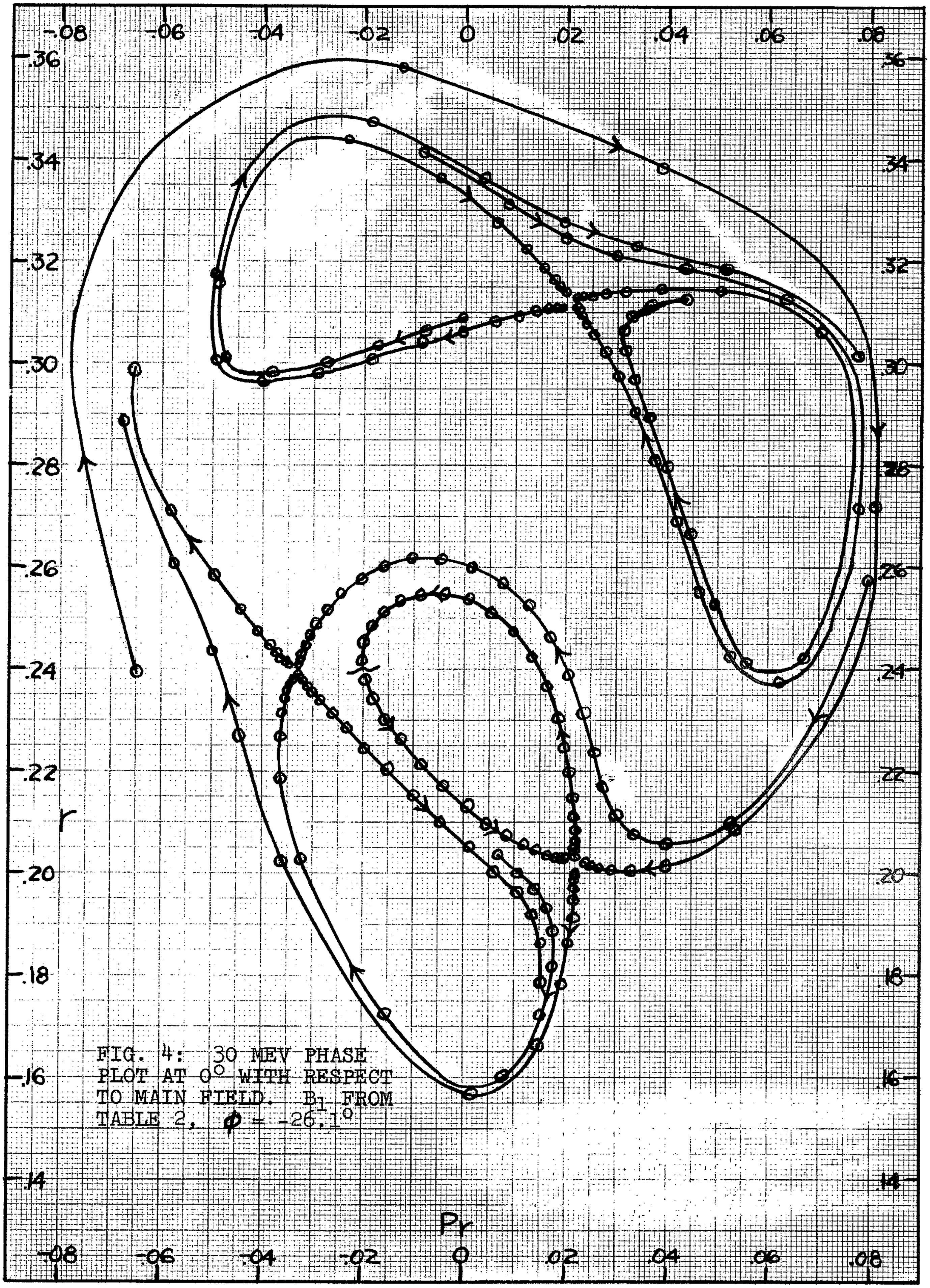

FIG. 4: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. B_1 FROM TABLE 2, $\phi = -26.1°$

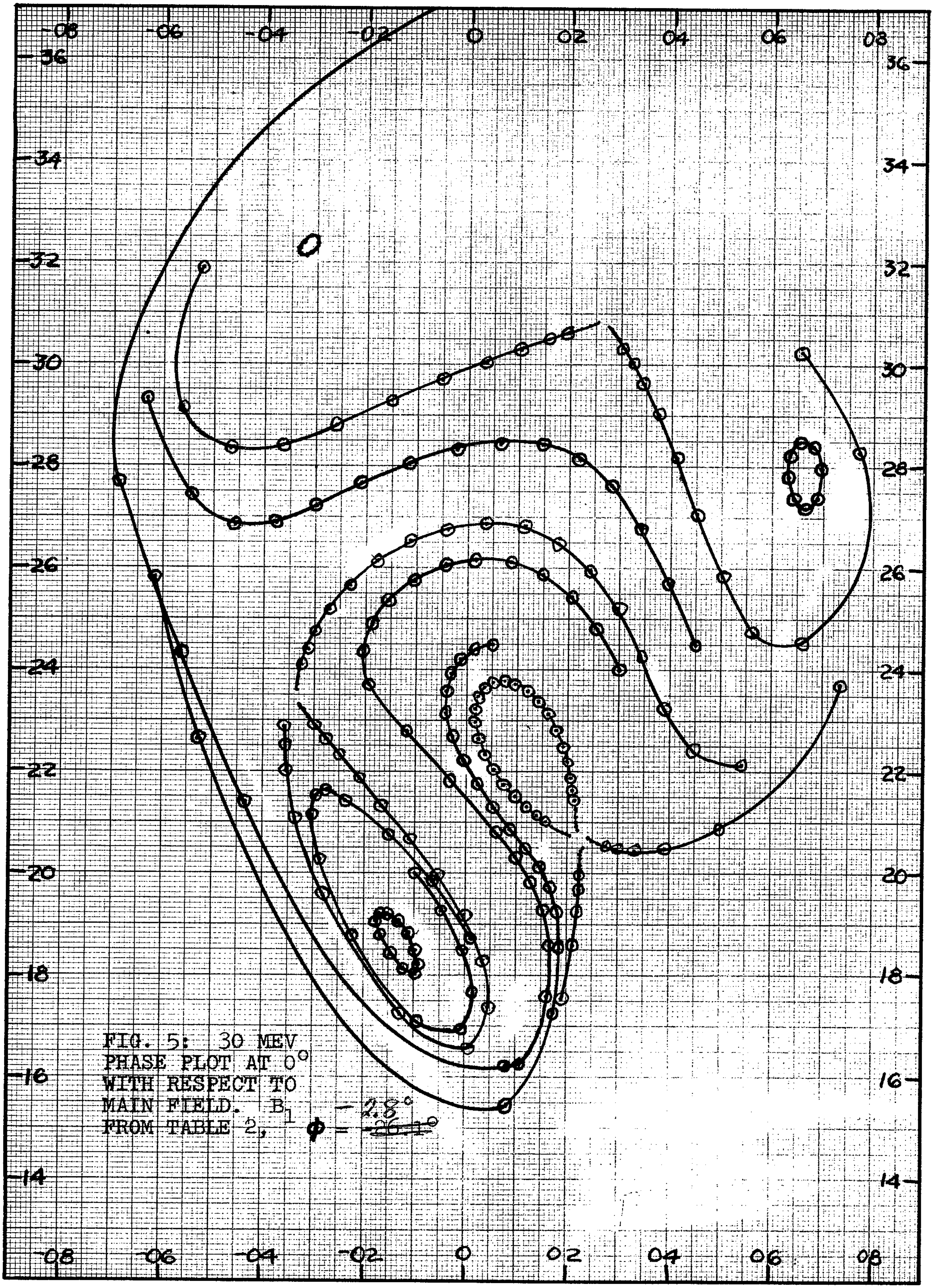

FIG. 5: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. B_1 FROM TABLE 2, ϕ = 2.8° ~~26.1°~~

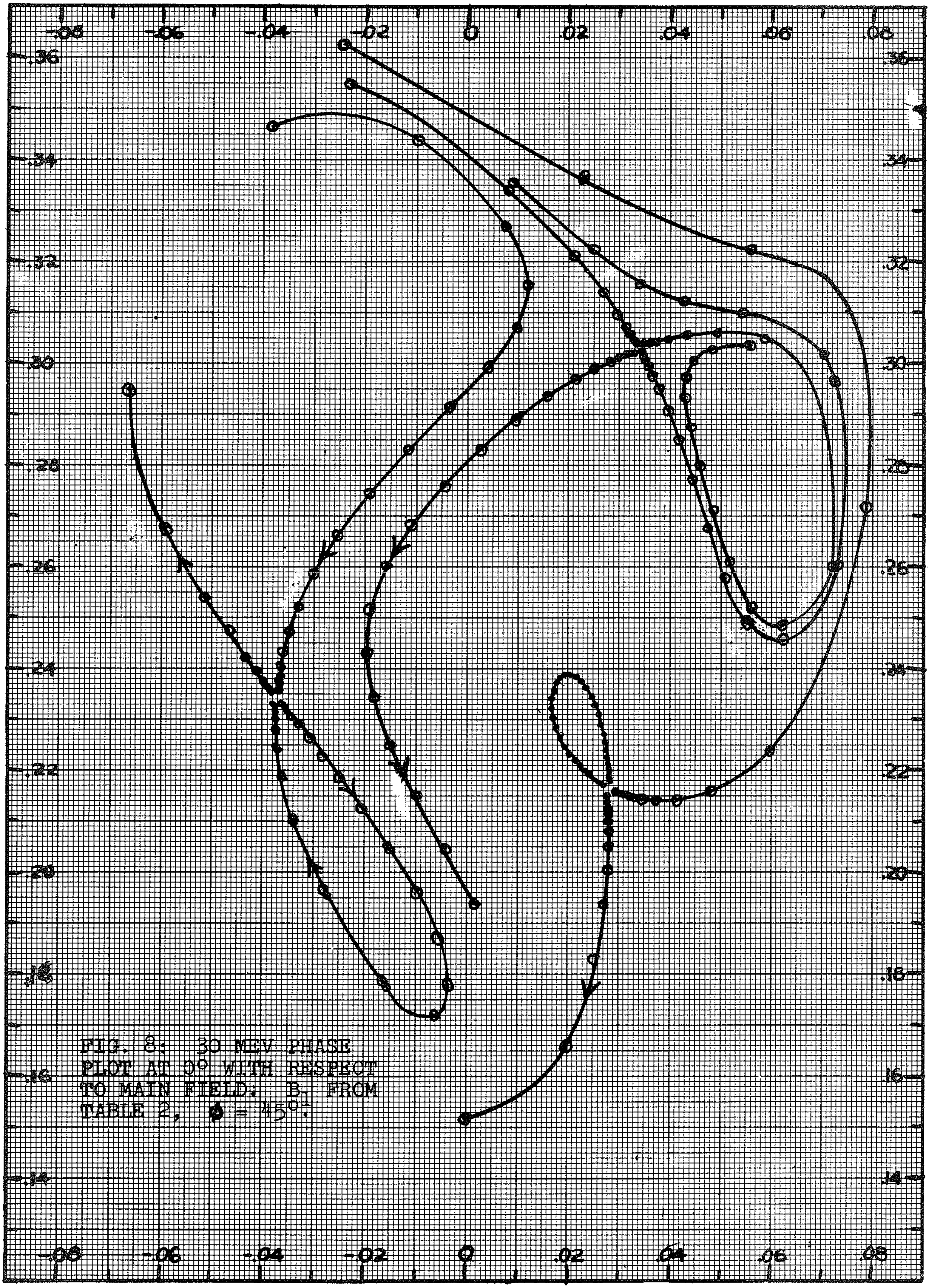

FIG. 8: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. B_1 FROM TABLE 2, $\phi = 45°$.

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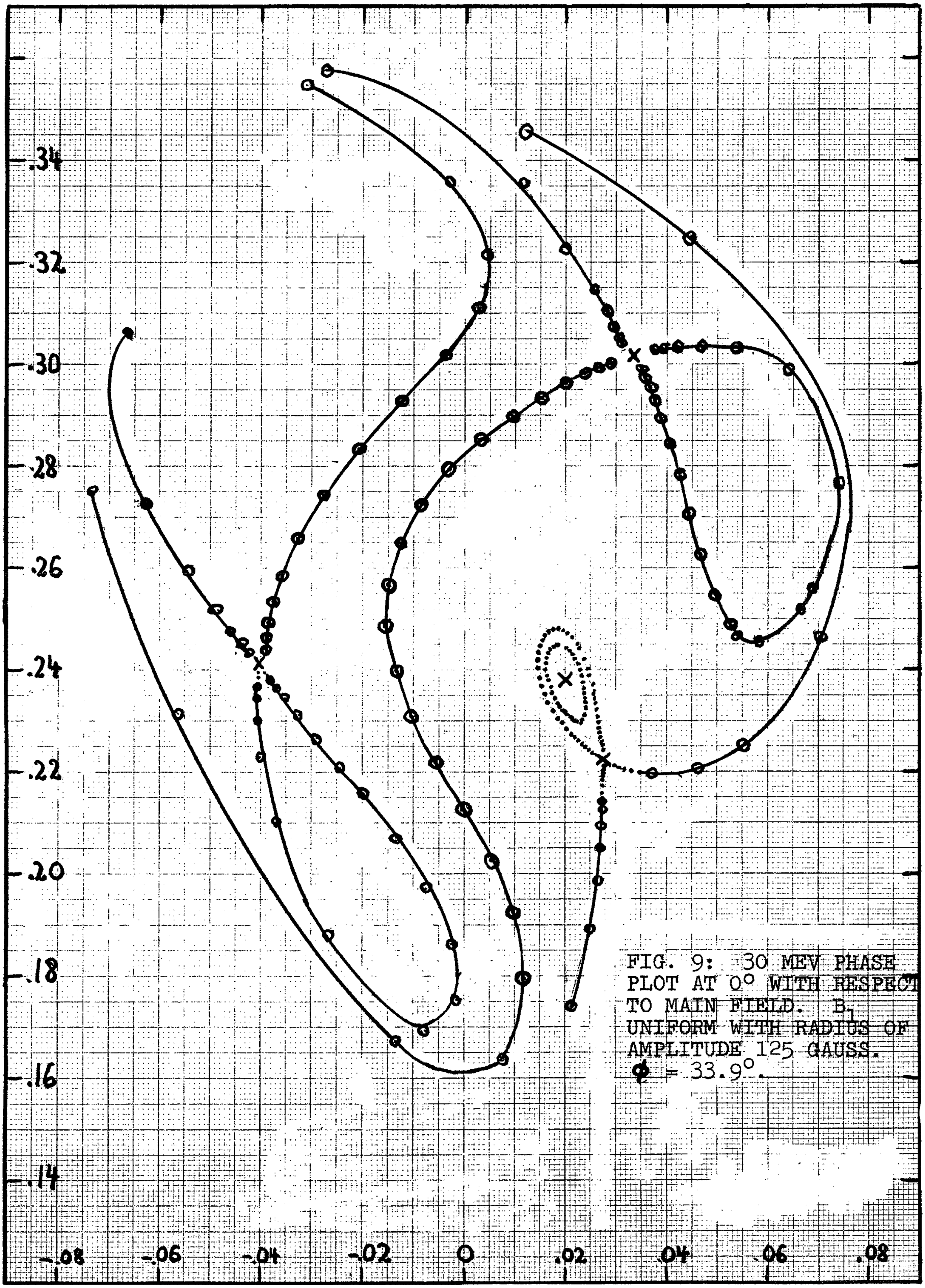

FIG. 9: 30 MEV PHASE PLOT AT 0° WITH RESPECT TO MAIN FIELD. B_1 UNIFORM WITH RADIUS OF AMPLITUDE 125 GAUSS. $\phi = 33.9°$.

points the trajectories interweave on the phase diagram as is to be expected in view of the general non-existence of integrals of the canonical equations.[5] All of the phase plots are at

5. Moser, J., Commun. Pure and Applied Math 8, 409 (1955)

$\Theta = 0^\circ$ with respect to the magnetic field, with radius plotted vertically in cyclotron units and radial momentum plotted horizontally in M_0C units.

IV. RELATION TO ORBIT SYMMETRIES

If the orbits corresponding to the three unstable fixed points are designated as U_1, U_2, and U_3 the approximate theory[1] yields five geometrical criterion for determining the cos Θ azimuth to yield a pure state (from the theory all five should be equivilant).

For a one corner opening at U_1 the five statements are:

(a) Set ϕ = angle at which 3Θ field is a minimum.

(b) Set ϕ = angle at which $P_r(U_1) = P_r(E.O.)$ and $r(U_1) < r(E.O.)$.

(c) Set ϕ = angle at which $r(U_2) = r(U_3)$ and $r(U_2) > r(E.O.)$

(d) Set $\phi = 180^\circ$ plus angle at which $P_r(U_1) = P_r(E.O.)$ and $r(U_1) > r(E.O.)$.

(e) Set $\phi = 180^\circ$ plus angle at which $r(U_2) = r(U_3)$ and $r(U_2) < r(E.O.)$.

For a pure two corner opening at $U_2 - U_3$ the criterion are the same except with maximum replacing minimum in (a) and with the inequality statements reversed in (b), (c), (d), and (e). All of the above statements with respect to r and P_r refer to the orbit positions in the absence of a cos Θ field.

Fig. 10 is a graph of r, P_r vs. Θ for the equilibrium

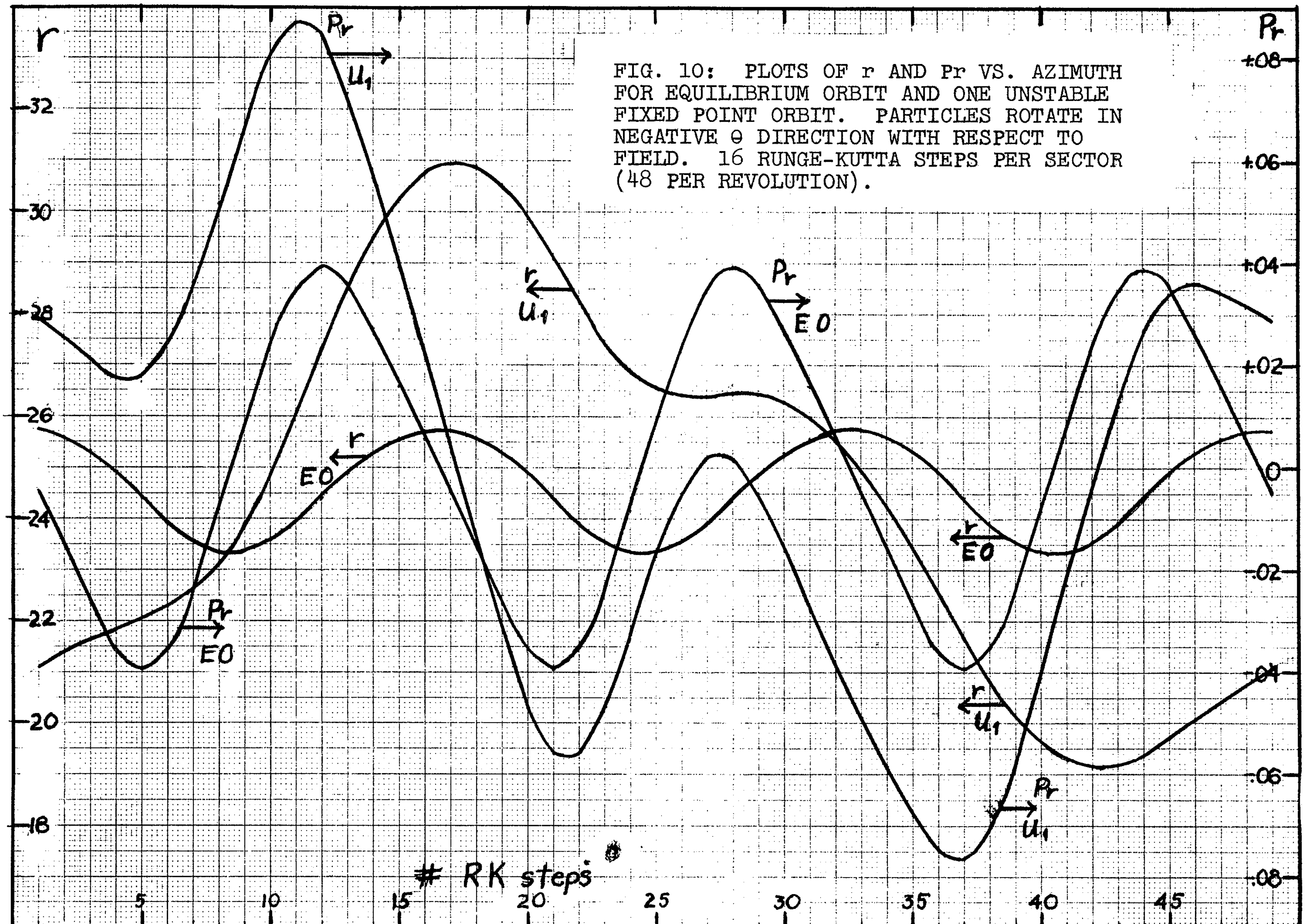

FIG. 10: PLOTS OF r AND Pr VS. AZIMUTH FOR EQUILIBRIUM ORBIT AND ONE UNSTABLE FIXED POINT ORBIT. PARTICLES ROTATE IN NEGATIVE θ DIRECTION WITH RESPECT TO FIELD. 16 RUNGE-KUTTA STEPS PER SECTOR (48 PER REVOLUTION).

orbit and for one of the unstable fixed point orbits in the field of table I and for a particle energy of 30 Mev. (Since the field has 120° symmetry the other two fixed point orbits can be obtained by shifting the one shown 120° forward and backward.)

With Table I and Fig. 10 the five criterion for ϕ can be evaluated; for a pure one corner opening the five choices for ϕ (in the same order as the criterion) are 4°, 21°, 36°, 43°, and 18°. For a pure two corner opening the five choices for are -56°, -39°, -24°, -17°, and -42°. The computer work as mentioned above indicates 32° and -37° to be the correct values. The results show marked scatter and little apparent reason for preferring any one of the criterion. (The first criterion is noticeably poorer. Since it is the only one of the five whose derivation assumes an unspiraled field, the result probably reflects effects of the spiral.) The marked difference between Figs. 7 and 9 indicates considerable interdependance of the angular position of $\cos \theta$ with its radial profile. This, in combination with the considerable scatter in the predicted values, implies that the criteria are of dubious value in guiding the placement of the $\cos \theta$ component in other fields than the one here considered; in such cases a certain amount of empirical searching similar to that conducted here will probably continue to be necessary.

V. ACKNOWLEDGMENT

It is a pleasure to acknowledge the important assistance of Mr. David A. Johnson in performing most of the computer studies herein described.