SECTION III

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# $\beta^{\dagger}$ Decay of <sup>20</sup>Na<sup>†</sup>

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The decay scheme of  $^{20}$ Na has been studied in detail to obtain information on the  $\beta$ -decay mirror symmetry properties of the mass-20 multiplet and to determine the Fermi decay strength to the T=1 isobaric analog state in  $^{20}$ Ne. The ratio  $ft^+/ft^-$  was determined to be  $1.026\pm0.024$  for the  $\beta^+$  transitions to the first excited state of  $^{20}$ Ne. The ft value for the transition to the T=1, 10.278-MeV level was measured to be  $2992\pm233$  sec indicating that most of the Fermi strength is concentrated in this transition. A weak  $\gamma$  transition from the  $2^+$  isobaric analog state to the  $2^+$  first excited state of  $^{20}$ Ne was observed and new  $\beta^+$ -delayed  $\alpha$  groups are reported at 3.27, 5.272, and 5.701 MeV. Theoretical ft values derived from the shell-model wave functions are compared with experiment.

#### I. INTRODUCTION

The  $\beta^+$  decay of  $^{20}$ Na has many interesting features which offer the possibility of carrying out a variety of experiments related to recent new ideas on meson exchange and second-class currents in the nuclear weak interaction. We have reported earlier our preliminary measurements on  $(\beta-\alpha)$  and  $(\beta-\nu)$  correlations for  $^{20}$ Na which are related to these studies  $^{1,2}$  and an extensive paper on this work will be published soon.

We report here our final results of experiments which were carried out to provide a set of ft values to test mirror symmetry in the mass-20 multiplet and to compare with recent shell-model calculations of ft values in the (0d-1s) shell. Earlier estimates of Wilkinson et al. for mass 20 showed a significant asymmetry  $(ft^*/ft^- = 1.054 \pm 0.023)$ . There were, however, some apparent errors in the original data on the  $\beta^+$  branching ratios of  $^{20}$ Na which Wilkinson corrected for by estimating experimental ft values based on assumptions of the Fermi strength to the analog state. The value reported earlier  $(\log ft = 3.77 \pm 0.010)^4$  was considerably larger than what had been expected for the analog state transition.

In order to obtain a precise number for the ft value of the transition to the first excited state of <sup>20</sup>Ne and to examine the details of the decay to the analog state, we have had to reinvestigate the complete decay scheme of <sup>20</sup>Na. Some of these results have been reported in short communications. <sup>5,6</sup>

In previous work on this nuclide, the first high-resolution measurement of the  $\beta$ -delayed  $\alpha$  spectrum was made by Macfarlane and Siivola. Additional details of the  $\alpha$  spectrum were later published by Polichar et al. and the first measurements of the  $\beta^+$  branching ratios were carried out by Sunier et al. Two determinations of the half-

life of  $^{20}$ Na have been recently reported  $^{1,3}$  which are approximately 10% larger than the older value of  $408 \pm 6$  msec.

## II. EXPERIMENTAL DETAILS

# A. Production and Source Preparation

High-intensity sources of  $^{20}$ Na (up to  $10^5$  dis/sec) were produced by the reaction  $^{20}$ Ne(p,n) $^{20}$ Na using 25-MeV protons from the Texas A& M cyclotron. The target was a mixture of neon and helium carrier gas. This gas mixture which also served as the carrier for the  $^{20}$ Na was continuously pumped from the target chamber through a 7-m-long, 0.7-mm-internal-diam Teflon capillary tube to a separate detection chamber. The  $^{20}$ Na source was produced in steady state by depositing the activity from the gas stream onto a collector. The carrier gas was pumped off using a high-speed vapor-booster pump which maintained the pressure near the source at 0.03 Torr. Details of the system are given elsewhere.

## B. α-Particle Spectrum Measurements

α-particle spectra were obtained using conditions that would give an optimum low background and high-energy resolution. A 50-mm2 Si(Au) surface-barrier detector with an energy resolution of 18 keV [full width at half maximum(FWHM)] for  $^{241}$ Am  $\alpha$  particles was used in the measurements. In order to reduce the very intense  $\beta$  background which is partly in coincidence with the  $\alpha$ spectrum we used two methods. First, a 20-kG magnetic field was maintained between the source and detector (1% geometry) to deflect the main component of the  $\beta$  radiation. In a second experiment, the  $\alpha$  detector was located behind a shielded collimator system at a geometry of 0.03% to minimize  $\beta$ - $\alpha$  pileup and to protect the detector from scattered radiation. For energy calibra-

# Isobaric Mass Quartets in the Mass-21 and Mass-37 Nuclei\*

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The  $^{40}$ Ca( $^{3}$ He,  $^{6}$ He) $^{37}$ Ca reaction has been used to measure the mass of  $^{37}$ Ca in its ground and first excited states. The mass of  $^{37}$ Ar and  $^{37}$ K in their lowest  $T=\frac{3}{2}$  states was also measured. The  $^{24}$ Mg( $^{3}$ He,  $^{6}$ He) $^{21}$ Mg reaction was used as a calibration, and consequently the energies of several excited states of  $^{21}$ Mg were measured. The mass-37 measurements give new precision values for the coefficients of the isobaric multiplet mass equation, and the excited states in  $^{21}$ Mg are members of quartets in the mass-21 nuclei.

# I. INTRODUCTION

In this paper we discuss quartets of  $T=\frac{3}{2}$  levels in the A=21 and 37 nuclei. The primary aim of the measurements was to reduce the errors of the masses of the quartet which includes the <sup>37</sup>Cl and <sup>37</sup>Ca ground states and their analogs in <sup>37</sup>Ar and <sup>37</sup>K. In the course of measuring the <sup>37</sup>Ca mass, several previously unobserved levels in <sup>21</sup>Mg were measured, and these are also discussed.

The dependence of the masses of the members of an isobaric quartet on its charge state is described by the isobaric multiplet mass equation (IMME). This equation, which is given by

$$M(T_z) = a + bT_z + cT_z^2$$

fits all the previously measured mass quartets remarkably well. The coefficients differ with the A and  $J^{\pi}$  of the levels in the quartet and are of interest for testing nuclear structure theories, since they measure both the Coulomb energy and its rate of change between isobars. In two of the completed quartets there is an indication that a small positive coefficient, d, for a cubic term is needed. These are the A=9 ground state<sup>1,2</sup> and the A=25 first excited state multiplet.<sup>3</sup>

# II. EXPERIMENTAL

# A. Nucleus of <sup>37</sup>Ca

The mass determination for <sup>37</sup>Ca was performed by comparing the rigidity of <sup>6</sup>He particles from the <sup>40</sup>Ca(<sup>3</sup>He, <sup>6</sup>He)<sup>37</sup>Ca reaction to the rigidity of <sup>6</sup>He's produced in the previously measured <sup>58</sup>Ni (<sup>3</sup>He, <sup>6</sup>He)<sup>55</sup>Ni <sup>4</sup> and the <sup>24</sup>Mg(<sup>3</sup>He, <sup>6</sup>He)<sup>21</sup>Mg<sup>1</sup> reactions. The experiment was performed in a split-pole spectrograph with 70.7-MeV <sup>3</sup>He particles from the Michigan State University sector-focused cyclotron. The position on the focal plane was determined by a resistive readout wire proportional counter. Particle identification was performed by time of flight relative to the cyclotron

beam structure. A plastic scintillator provided time-of-flight and total energy information which was essential to the identification of the <sup>6</sup>He particles. The experimental arrangement is described in more detail in a paper on the <sup>13</sup>C(<sup>3</sup>He, <sup>6</sup>He)<sup>10</sup>C reaction.<sup>5</sup>

The targets which were used in the present experiment were natural Ca, one self-supporting  $2.7 \text{ mg/cm}^2$  and the other  $1.0 \text{ mg/cm}^2$  on a 50-mg/cm<sup>2</sup> carbon backing. Relatively thick targets are required because of the cross section ( $\sim 0.2 \ \mu b/sr$ ) of the reaction, which is small even when compared to other (3He, 6He) reactions. The calibration reactions on <sup>58</sup>Ni and <sup>21</sup>Mg, for example, have 5 to 10 times greater cross sections. The thickness of the targets used was determined both before and after the measurements by observing the energy loss of  $^{241}\mathrm{Am}~\alpha$  particles in the targets. Spectra from <sup>58</sup>Ni and <sup>24</sup>Mg and the thicker of the two Ca targets are shown in Fig. 1. The three spectra were taken with magnetic fields calculated to place the ground-state 6He particles at the same location (channel number) on the focal plane. The proton NMR resonant frequencies given on the figure correspond to these three fields calculated from previous measurements of the 55Ni, 37Ca, and 21Mg masses. A peak corresponding to the previously observed<sup>6</sup> first excited state of <sup>37</sup>Ca is seen, as is the broadening and shifting of the 37Ca ground-state peak compared to the other two ground-state peaks. The broadening is due to the thicker target, but most of the shift is an indication of the 90-keV difference between the present and previous measurements. A comparison of the present to the previous measurement of <sup>37</sup>Ca is given in Table I. The error of the 37Ca mass measurement is dominated by the uncertainties in the Q values of the calibration reactions and by the uncertainty in thickness corrections. Errors in the determination of the magnetic fields needed to bend the highly rigid <sup>6</sup>He particles also contributed to the error.

# Shell-Model Study of <sup>24</sup>Ne<sup>†</sup>

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Shell-model calculations for the A=24, T=2 system are presented which use an empirically modified version of Kuo's realistic two-body interaction and a large, but truncated, sd-shell basis space. Using the same two-body matrix elements, single-particle energies and effective charges that were found to be successful in the full sd space for A=19-22, good agreement with experimental measurements is obtained for  $^{24}$ Ne levels and other members of the A=24, T=2 system. Some predictions are made for the unreported isotope  $^{24}$ Si.

#### I. INTRODUCTION

This paper describes a large-basis shell-model calculation for the A = 24, T = 2 system, with particular consideration of 24Ne 14. Although 24Ne is difficult to study experimentally, there is now a significant body of data obtained via the  $^{22}Ne(t, p)$ -<sup>24</sup>Ne reaction, <sup>1-3</sup> the <sup>22</sup>Ne $(t, p_{\gamma})$ <sup>24</sup>Ne reaction, <sup>2, 3</sup> and the  ${}^{3}H({}^{22}Ne, p_{\gamma}){}^{24}Ne$  reaction. 4, 5 Efforts to understand the 24Ne spectrum in terms of projected Hartree-Fock<sup>6</sup> (HFP) and projected Hartree-Bogoliubov (HBP) calculations have not been successful. It is thus of interest to see whether, as in the case of 24Mg,8 a shell-model calculation can reproduce simultaneously the collective and the individual-particle aspects of this nucleus. The main sections of the paper describe the calculation and diagonalization of the Hamiltonian matrices and the calculation of energy eigenvalues and electromagnetic properties. Two-nucleon transfer, and the  $\beta$  decays of <sup>24</sup>Ne and the unreported isotope <sup>24</sup>Si are also discussed briefly.

# II. CALCULATION OF HAMILTONIAN

The two-body matrix elements used in this work are those described by Preedom and Wildenthal9 and used in shell-model calculations for A = 19-22, and 23 and 24.8 They are based on Kuo's realistic effective interaction, 10 but have been modified as described in Ref. 9 to produce better agreement of shell-model eigenvalues (calculated in the full sd-shell basis) with experimental level energies in the A=18 to 22 mass region. No data from nuclei with A > 22 were used in the adjustment. Single-particle energies, taken directly from the 17O spectrum, were equal to -4.15, -3.28, and +0.93MeV for the  $0d_{5/2}$ ,  $1s_{1/2}$ , and  $0d_{3/2}$  orbitals, respectively. Although all three sd-shell orbitals were allowed to be active, it was necessary in the present calculation to restrict occupancy of the dorbitals in order to keep the dimensionality of the basis space manageable. Specifically, no fewer

than four particles were permitted in the  $0d_{5/2}$  shell, and no more than two in the  $0d_{3/2}$  shell. With these restrictions, the dimensions of the largest matrix to be diagonalized were  $442 \times 442$  for J=3.

The diagonalization itself was carried out at the Oak Ridge National Laboratory on an IBM 360 series computer using the Oak Ridge-Rochester shell-model code, 11 and the calculation of observables from the wave functions was done on the XDS Sigma-7 computer of the Michigan State University Cyclotron Laboratory.

The calculated energy spectrum for  $^{24}$ Ne is shown in Fig. 1 compared to the experimental spectrum as given by Howard  $et\ al.^3$  Also shown are the calculations of Khadkikar, Nair, and Pandya<sup>6</sup> and Goeke, Faessler, and Wolter.<sup>7</sup> There is good agreement between the shell-model spectrum and all known states of  $^{24}$ Ne. In fact the correspondence is sufficiently close that one may assign the 4.89-MeV state a probable spin and parity of  $3^+$ . Such an assignment is in accord with the experimental observation that the state is relatively weak in (t,p), as would be expected for an unnatural parity transition.

The calculation for the T=0 states of <sup>24</sup>Mg with the same Hamiltonian has been described previously,8 and there is good agreement with experiment for all but a few levels. However, when the positions of the lowest T=1 and T=2 levels in <sup>24</sup>Mg are calculated, the predicted excitation energies are too low by a few hundred keV for T=1 and 2 MeV for T=2. The source of the discrepancy is apparently the truncation of the basis space. For a few spins, J=0 and  $J \ge 8$ , eigenvalues have been recalculated in the full sd basis, and the observed T=0 to T=2 splitting is then reproduced almost exactly (Fig. 2). Furthermore, the positions of excited 0+ and 8+ states do not change significantly relative to the lowest 0<sup>+</sup> state for each value of T. Thus there is some reason to believe that the effect of basis truncation is to shift the entire spectrum for a given value of T without greatly

# Production of the Light Elements Lithium, Beryllium, and Boron by Proton-Induced Spallation of <sup>14</sup>N<sup>†</sup>

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Astrophysically interesting cross sections for the production of isotopes of lithium, beryllium, and boron in the proton-induced spallation of <sup>14</sup>N were measured for proton energies between 17 and 42 MeV. A time-of-flight method was used for mass identification, supplemented at one energy by radioactivity analysis. Gas targets were used throughout. The astrophysical significance of the results is discussed.

#### I. INTRODUCTION

The astrophysical mechanisms responsible for creation of the light elements Li, Be, and B are still uncertain in detail. Since these light elements do not survive the hydrogen-burning stage in stars, and are not completely accounted for by more primeval nuclide formation, and the most likely sites for their origin are galactic cosmic rays for stellar surfaces. In these environments, proton- or  $\alpha$ -induced spallation of the more abundant targets, chiefly  $^{12}$ C,  $^{14}$ N,  $^{16}$ O,  $^{20}$ Ne,  $^{24}$ Mg, and  $^{28}$ Si is expected to be the dominant source of Li, Be, and B.

To calculate their production from models of the process, one needs the spallation cross sections for all probable targets. The cross sections for <sup>14</sup>N are particularly important because the reaction thresholds are low compared to those for other likely targets, and the proton fluxes encountered in nature are peaked at low energies. <sup>6,7</sup> Thus production from <sup>14</sup>N is favored over that from other targets. However, only a few measurments of the <sup>14</sup>N cross sections are available and with the exception of a single measurement<sup>8</sup> in emulsions at 125 MeV, these are only for the radioactive products <sup>7</sup>Be and <sup>11</sup>C.

As part of a program<sup>9,10</sup> of measuring production cross sections for the light elements in the proton energy range accessible with the Michigan State University (MSU) sector-focused cyclotron, we performed cross-section measurements on <sup>14</sup>N for protons with energies between 17 and 42 MeV. The experimental techniques are described in Secs. II and III; the resulting production cross sections are presented and discussed in Sec. IV.

# II. TIME-OF-FLIGHT MEASUREMENTS

## A. Detection Method

The major difficulty encountered in measuring spallation cross sections in this energy range is

that the bulk of the mass 6–11 reaction products have energies well below 10 MeV and it is not possible to distinguish among them by using standard  $\Delta E$ -E particle-identification techniques. Other methods such as identification by radioactivity or mass spectrometry suffer from the disadvantage of not being equally sensitive to all isotopes. In most of the present measurments mass identification was accomplished using a time-of-flight method similar to that previously described, 10 but adapted for the use of gas targets. Only a brief description will be given here; details may be found in Ref. 10.

The time-of-flight method is based on the following principle. Reaction products of mass m originate in the target at time intervals precisely correlated with the cyclotron radiofrequency. After traversing a flight path d they are stopped in a detector which yields a timing pulse as well as a signal proportional to energy E. The flight time tof the particle is thus established (to within a multiple of rf). The quantity  $Et^2 = md^2/2$  then identifies the mass of the detected particle. Ascertaining only the mass number may seem a drawback but for our astrophysical application it is quite sufficient since only one isobar per mass number is stable in the region  $6 \le A \le 11$ . All other isobars of the same A detected in our experiment decay to it in a time short compared to astrophysical time scales with the two minor exceptions of <sup>9</sup>C and <sup>10</sup>Be. These are not expected to affect the interpretation of our measurements for reasons we shall discuss in Sec. IV. Thus, the cross sections we measure for a given mass can be identified, for most astrophysical purposes, with a particular isobar as follows: mass 6 = 6Li, mass  $7 = {}^{7}\text{Li}$ , mass  $9 = {}^{9}\text{Be}$ , mass  $10 = {}^{10}\text{B}$ , and mass  $11 = {}^{11}B$ .

### B. Gas Target

There are two advantages in using as a target nitrogen gas rather than some solid compound con-

# High-Resolution Study of ${}^{48}\text{Ca}(p,t){}^{46}\text{Ca}$ at $E_p = 39 \text{ MeV*}$

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The  $^{46}$ Ca nucleus was studied using the (p,t) reaction. A number of spin-parity assignments are clarified for the low-lying states. Many new energy levels are reported up to an excitation energy of 6.3 MeV including an additional  $0^+$  state at 4.76 MeV. Distorted-wave calculations with f-p shell wave functions give reasonable agreement for the first  $0^+$ ,  $2^+$ ,  $4^+$ , and  $6^+$  states and for two weak  $0^+$  states near 5.5 MeV which were strongly excited in the  $^{44}$ Ca(t,p) reaction.

## 1. INTRODUCTION

There are still many difficulties in using twoneutron-transfer reactions to check detailed nuclear wave functions. One useful technique is to compare the cross sections for the excitation of the same states, particularly 0° states, in the same final nucleus reached by both (p, t) and (t, p)reactions. 1 This method was recently used by Broglia, Kolltveit, and Nilsson<sup>2</sup> in the calcium isotopes to study 0+ states. Such a study would benefit by more precise data on the  ${}^{48}\text{Ca}(p,t)$  reaction since the complementary reaction  $^{44}Ca(t,p)$ has already been carried out by Bjerregaard et al.3 In this stripping experiment a number of 0+ states were observed between 5 and 6.5 MeV with cross sections from 9 to 55% of the ground-state cross section. In the (p, t) experiments on  ${}^{48}$ Ca  ${}^{4-7}$  to date, the energy resolution has not been sufficient to extract these 0+ states from the high density of states above 5 MeV. In addition the spin assignments for a number of states below 5 MeV from the  $^{48}$ Ca(p, t) experiments did not agree with those from the  $^{44}$ Ca(t, p) experiment. Finally another (t,p) experiment<sup>8</sup> on <sup>44</sup>Ca with lower resolution suggested different 0+ states above 6 MeV. It was therefore decided to carry out the  $^{48}$ Ca(p, t) experiment to check these discrepancies and to compare with the  $^{44}Ca(t,p)$  experiment. A preliminary report of this work has been presented elsewhere.9

# II. EXPERIMENT

The experiment used the 39-MeV proton beam from the Michigan State University cyclotron. This gave a maximum triton energy of about 30 MeV, the maximum rigidity particle which could be bent by the Enge split-pole spectrograph. The experiment was performed in two stages which nicely complemented each other. In one case, a single-wire proportional counter was used in the focal plane of the Enge spectrometer with a 1.1mg/cm<sup>2</sup> self-supporting <sup>48</sup>Ca target (96.3% <sup>48</sup>Ca) obtained from the Oak Ridge National Laboratory. The energy resolution, primarily limited by the target thickness, was about 50 keV. However, this arrangement allowed very accurate angular distributions to be obtained for a number of the strong low-lying excited states from 3 to 70° in the laboratory. Special care was taken to trace out the deep minima, particularly for the 0+ angular distributions.

A high-resolution study was also carried out in which the tritons were detected in Kodak NTB photographic emulsions. In this case a  $50-\mu g/cm^2$ <sup>48</sup>Ca target was used. This metallic target was prepared from the carbonate by vacuum evaporation from a tantalum tube. The calcium (enriched to 97.2% <sup>48</sup>Ca) was deposited on a 20- $\mu g/cm^2$  carbon foil which was supported by one layer of Formvar. The target was stored and transferred to the scattering chamber under vacuum. The resolution of the system was optimized by observing tritons in the focal-plane speculator system. 10 A total resolution of 11 kev full width at half maximum was obtained. The plate data were not as extensive as those from the proportional counter because of the limitation of scanning many plates, and extended from 10 to 50° in 4° steps.

# III. RESULTS

A spectrum of tritons from the photographic plate data is shown in Fig. 1. The regions around 3.6, 4.4, and 4.7 MeV are also shown expanded to better illustrate the doublet structure. (See Fig. 2.) Note also the state at 3.987 MeV which has not been observed previously. No evidence is seen at any angle for the state at 3.780 MeV observed in an inelastic scattering experiment on 46 Ca. 11 Many more states were observed above 5 MeV than had been observed previously.

A list of states including energies and spin-pari-

# Isospin Mixing from the Effective Nucleon Interaction\*

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Effective interactions determined from closed-shell-plus-two-particle spectra have a charge dependence which predicts isospin mixing for more complicated nuclei. We infer the isospin mixing in A=44, 46, 48, and 52 on the basis of the observed A=42 spectra. Agreement with experimental data is reasonable in four out of six cases; the best example is  $^{44}$ Sc, where we verify that the n-p effective residual interaction is stronger than the n-n interaction. We also compare predictions to experiment for isospin mixing in A=20 and 24, based on the charge dependence seen in the A=18 spectrum.

## 1. OBJECTIVES

The charge dependence of the nuclear interaction is currently under vigorous study. Not only is the subject of intrinsic interest, but recent studies of Coulomb energies of nuclei have shown that the pure Coulomb interaction, taken together with reasonable models for the nuclear wave functions, is unable to account for energy differences of mirror nuclei.1-4 The discrepancy is systematic in sign and ranges in magnitude from 100 keV in the mass-3 doublet to about 1 MeV in mass 208.2 In the hope of understanding the empirical charge dependence of the interaction better, we examine in this paper  $\beta$ -decay properties of nuclear states which are sensitive to charge dependence. In general, nuclear-structure observables which can be studied include level shifts in the spectra of isospin multiplets, and certain isospin impurities in the physical states.

Our starting point is a limited shell-model basis and an effective charge-dependent interaction determined from the simplest spectra in that basis, i.e., the one-particle and the two-particle nuclei. We apply this interaction to calculate the wave functions of heavier nuclei. Due to the charge dependence, isospin will be mixed in these heavier nuclei. This is observed experimentally by isospin-forbidden Fermi admixtures in Gamow-Teller  $\beta$  decays. In the next section we discuss the interactions in our model spaces. We also briefly describe the extraction of the  $\beta$ - $\gamma$  circular polarization asymmetry, an experimentally measured quantity, from the theoretical-model wave functions. Theory and experiment are compared for nuclei in the  $f_{7/2}$  shell and the sd shell. We conclude that there is definite evidence in the effective interaction for a stronger np force, at least in the  $f_{7/2}$  shell. However, whether this is fundamental or a result of the shell truncation is unclear. In most cases, the shell description of the nuclei is not accurate enough to distinguish Coulomb from

non-Coulomb sources of isospin mixing.

Previously, calculations of isospin impurities relating to isospin-forbidden  $\beta$  decay were made by Blin-Stoyle and others. 5-8 Our calculations are superior in the following respects: Since twoparticle empirical spectra are now available, we do not have to resort to a multiparameter description of the charge-dependent interaction. Without the charge dependence as a free parameter, agreement or disagreement checks the validity of the whole shell-model approach. Also, we have better nuclear wave functions than were available previously. This is made possible by the Oak Ridge-Rochester shell-model code9 and the extensive exploration of interactions and the consequences for observables made for the sdshell nuclei.

# 2. CHARGE-DEPENDENT INTERACTION

The evidence on the charge dependence of the nuclear force comes most fundamentally from few-body-scattering data. A comprehensive review has been given by Henley.10 One firm conclusion from the data on scattering lengths is that the neutron-proton interaction is about 2% stronger than either the neutron-neutron interaction or the non-Coulomb part of the proton-proton interaction. Unfortunately, the scattering lengths depend on the depth and range of the potential in the combination (depth) $\times$  (range)<sup>2</sup>, while the properties of bound states are most sensitive to the potential depth and range in the combination (depth)  $\times$  (range)<sup>3</sup>. Thus an accurate measurement of the ranges of the potentials is also needed, and this is not yet available with adequate precision. The data are consistent either with an interaction having a charge-independent volume integral and/or with an interaction 2% stronger in the neutronproton system.

To determine the charge dependence of the shellmodel interaction, we turn to the spectra of nuclei that we can describe simply in the shell model.

# Shell-Model Calculations for A = 6-14 Nuclei with a Realistic Interaction\*

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Shell-model calculations are performed on the normal parity states of 0p-shell nuclei with A=6-14. The Hamiltonian is diagonalized in the full 0p basis, and the effective two-body interaction is computed from the Sussex relative harmonic-oscillator matrix elements. The second-order corrections to the two-body matrix elements are calculated for all intermediate states up to  $2\hbar\omega$  excitation energy. The harmonic-oscillator size parameter is taken to be constant at 1.7 fm for all nuclei, and the  $p_{3/2}$ - $p_{1/2}$  single-particle energy splitting is determined for each mass number by a least-squares rms fitting to the experimental spectrum. Static and dynamic properties of the energy levels are calculated and found to be usually in good agreement with experiment.

#### 1. INTRODUCTION

The basic problem in any feasible shell-model calculation is to determine the effective two-body interaction appropriate for the chosen configuration space. The most appealing method is, of course, to compute this interaction directly from the free nucleon-nucleon scattering phase shifts, without introducing any parameters. The pioneering work of Kuo and Brown, 1-3 using the Hamada-Johnston potential, has yielded very promising results, and today a number of successful shell-model calculations, based on the Kuo-Brown interaction, exist for several nuclear regions. 4,5

An interesting alternative to the Kuo-Brown method has been suggested by Elliott and his collaborators at Sussex.6,7 By making some reasonable assumptions about the smoothness and range of the potential, they are able to deduce the relative harmonic-oscillator matrix elements directly from the phase shifts without ever constructing an explicit form for the interaction. In spite of a number of applications with the Sussex matrix elements (see, e.g., Ref. 8), they have rarely been used in any detailed shell-model calculations. In the present work, we shall apply an effective Sussex interaction to the normal parity states of the 0p shell, computing not only energy levels, but also numerous other observables which have been measured and exist in the literature.

Our primary reason for choosing the 0p shell is that a comprehensive investigation of these states, employing realistic two-body forces, has so far not been performed. One previous calculation, using an effective Hamada-Johnston potential, was carried out in this region by Halbert, Kim, and Kuo, but they computed only the energy levels. Some qualitative comparisons with this work will be given in the last section. We shall also compare our results with the successful work

of Cohen and Kurath, <sup>10</sup> who determined their twobody effective interaction by a least-squares fitting of up to 17 free parameters. Even though their method for obtaining an effective interaction is the opposite of ours, it will be shown in the present work that most of our computed observables are very similar to theirs.

Other shell-model calculations have been performed in this region and must be briefly mentioned at this time. For example, the intermediate coupling calculations<sup>11,12</sup> yield quite satisfactory results provided one allows the spin-orbit term to gradually increase throughout the shell. Also, the least-squares technique of Cohen and Kurath has been extended by Goldhammer and coworkers<sup>13</sup> to include some three- and four-body effective forces. In most cases, very good agreement with experiment is obtained.

In Sec. II we present the necessary theory for constructing the Hamiltonian matrices and for computing the effective two-body matrix elements (2BME's). The theoretical energy-level fittings for each nucleus are then presented in Sec. III. We further test the wave functions in Sec. IV by comparing radiative transitions, spectroscopic factors for one-nucleon-transfer reactions, static dipole moments, and  $\beta$ -decay rates with experiment. Finally, in Sec. V we discuss the results and present comparisons with other shell-model calculations.

# II. THEORY

The construction and diagonalization of the energy matrices were performed with the Oak Ridge-Rochester shell-model codes, <sup>14</sup> which require the two-body matrix elements to be expressed in the *jj*-coupling scheme. The "bare" two-body matrix elements corresponding to the first diagram of Fig. 1 have been computed from the relative harmonic-oscillator matrix elements tabulated in

# Mass of 31S†

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A high-resolution study of the  ${}^{32}\mathrm{S}(p,d){}^{31}\mathrm{S}$  reaction shows a discrepancy with the current table of mass values. The  ${}^{32}\mathrm{S}(p,d){}^{31}\mathrm{S}$  reaction Q value is found to be - 12 817.8  $\pm$  1.5 keV, 45.6 keV more positive than the published value, which carried an 11-keV uncertainty. Similar studies of the (p,d) reaction on  ${}^{23}\mathrm{Na}$ ,  ${}^{27}\mathrm{Al}$ ,  ${}^{33}\mathrm{S}$ ,  ${}^{34}\mathrm{S}$ ,  ${}^{35}\mathrm{Cl}$ , and  ${}^{37}\mathrm{Cl}$  indicate no discrepancies with the Q values (of 1- to 2-keV accuracy) tabulated for these nuclei. Possible sources for the sulfur discrepancy are discussed.

NUCLEAR REACTIONS  $^{23}$ Na,  $^{27}$ Al,  $^{32}$ ,  $^{33}$ ,  $^{34}$ S,  $^{35}$ ,  $^{37}$ Cl (p,d), E=35 MeV, measured Q, deduced new value for the mass of  $^{31}$ S.

In a recent letter, Goss, Browne, and Rollefson reported a new value for the SNi(p,  $\alpha$ )SCO reaction Q value which disagrees with the number calculated from the 1971 Wapstra-Gove Mass Tables by several standard deviations. We report here an error in one of the tabulated mass values for the s-d shell region which is of a comparable magnitude to that found in the iron region. In a high-resolution study of the (p,d) reaction on sulfur we find the  $^{32}$ S(p, d) $^{31}$ S Q value to be  $45.6\pm1.5$  keV more positive than is indicated by the 1971 Mass Table. The estimated uncertainty of the previous value was 11 keV.

In our experiments, we used a 35-MeV proton beam extracted from the Michigan State University Cyclotron. The beam at the target had a coherent energy spread of about 20 keV. The use of dispersion matching and the other techniques described by Blosser et al.3 enabled us to obtain a resolution of about 8 keV, full width at half maximum, for 25-MeV deuterons at the focal plane. The sulfur target (84.5, 0.5, and 15% of  $^{34}S$ ,  $^{33}S$ , and  $^{32}$ S, respectively) was about 10  $\mu$ g/cm<sup>2</sup> thick and was sandwiched between thin carbon foils and Formvar films. Spectra of the reaction products were recorded on two abutting nuclear emulsion plates spanning 50 cm of an Enge-type magnetic spectrograph focal plane. Deuteron and proton groups from the (p,d) and (p,p) reaction on the intended target nuclei and on various contaminant nuclei also present in the target were identified on the basis of a first-order spectrograph calibration and/or on differences in particle-track ionization densities. These identifications were then checked by a comparison of final precise excitation-energy assignments at several different angles of observation.

Precise energy values for all observed particle groups were obtained by slightly adjusting about their nominal values the various parameters which

affect the calculated values of emergent-particle momenta in our reaction-particle kinematics program, so as to obtain a least-squares fit of the excitation energies (total Q values) of selected reference states to their accurately known values. Parameters which were varied in this procedure were the beam energy, angle of observation, gap between abutting plates, and the linear and quadratic parameters of the Bp vs focal-plane-position relationship for the spectrograph. For reference peaks in the sulfur spectra, for example, we used deuteron groups from the ground-state transitions of the  $^{34}$ S,  $^{28}$ Si,  $^{16}$ O,  $^{14}$ N, and  $^{12}$ C(p,d) reactions, and proton groups from elastic scattering on 12C and 16O and from inelastic scattering to the first excited states in 32S and 34S. The inclusion of both deuteron and proton groups, and reactions on a significant range of target masses, makes an accurate determination of the beam energy and scattering angle possible. The inclusion of (p, d)groups leading to several low-lying excited states of  $^{33}$ S, whose excitation energies are known to  $\pm 1$ keV accuracy from Ge(Li) detector studies of their  $\gamma$ -ray decays, leaves the results of the adjustment for these sulfur data essentially unchanged. Finally, the Q value of the reaction in question, <sup>32</sup>S- $(p,d)^{31}$ S, was adjusted from its nominal value until the particle groups corresponding to the ground state and the excited states (when accurate values of their excitation energies were available) of the residual nucleus were matched to their observed positions.

The uncertainties which reside in this procedure were estimated from trials with several different target nuclei, from trials for a particular target with several different combinations of input "known-energy" particle groups, and from trials with the same "reference data set" at several different angles of observation, again for a particular target. The standard deviation in the assigned

# High-Resolution Study of the Particle-Hole Multiplets in <sup>208</sup>Bi<sup>†</sup>

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Particle-hole multiplets in  $^{208}$ Bi corresponding to the coupling of the  $h_{9/2}$  proton with neutron holes in the  $p_{1/2}$ ,  $f_{5/2}$ ,  $p_{3/2}$ ,  $i_{13/2}$ ,  $f_{7/2}$ , and  $h_{9/2}$  shells have been observed by the  $^{209}$ Bi (p,d)- $^{208}$ Bi reaction at 35 MeV with an over-all resolution of 5 keV in the deuteron spectra. The results generally show excellent agreement with the weak coupling model except for the  $h_{9/2}$  hole states. However the observed  $h_{9/2}$  multiplet states do contain a large fraction of the expected strength. There is also substantial agreement with earlier (d,t) work except for some spin assignments and in the location of the  $2^-$  member of the  $\pi h_{9/2}$ ,  $\nu^{-1} i_{13/2}$  multiplet.

NUCLEAR REACTIONS <sup>209</sup>Bi(p, d), E=35 MeV; measured levels,  $\sigma(\theta)$ , deduced S.

#### I. INTRODUCTION

Recent discussions of the effective two-nucleon force obtained from the two-particle spectra of nuclei near closed shells¹ has again focussed attention on such nuclei. One important example near the doubly magic nucleus  $^{208}\mathrm{Pb}$ , is the nucleus  $^{208}\mathrm{Bi}$  with a proton outside the  $^{208}\mathrm{Pb}$  core and a neutron hole in the core. In the present experiment, we have studied  $^{208}\mathrm{Bi}$  by the reaction  $^{209}\mathrm{Bi}$ - $(p,d)^{209}\mathrm{Bi}$  using the high-resolution capability of the Michigan State University (MSU) cyclotron. In the simplest picture of this reaction, we expect to reach states which consist primarily of a proton in the  $h_{9/2}$  orbit coupled to neutron holes in the  $2p_{1/2}$ ,  $1f_{5/2}$ ,  $2p_{3/2}$ ,  $0i_{13/2}$ ,  $1f_{7/2}$ , and  $0h_{9/2}$  orbits. A number of previous studies $^{2-4}$  have investigated

A number of previous studies<sup>2-4</sup> have investigated the low-lying levels in <sup>208</sup>Bi. In particular the single-particle transfer reactions <sup>209</sup>Bi(d, t)<sup>208</sup>Bi and <sup>207</sup>Pb(<sup>3</sup>He, d)<sup>208</sup>Bi have indicated the simple particle-hole multiplet structure of these states. This feature is also common to the theoretical descriptions of this nucleus, <sup>5.6</sup> at least for the low-lying multiplets.

The present experiment extends the previous work with the better resolution available with the MSU cyclotron facility and checks the spin-parity assignments from the (d,t) reaction. A further motivation for the experiment was the fact that the angular distributions for different angular momentum transfers in the (p,d) reaction at 35 MeV are quite unambiguous. This allows easy identification of the members of a multiplet and serves as an indication of any mixture of different l values in a particular transition.

Finally a careful comparison was made of the present  $^{209}$ Bi $(p,d)^{208}$ Bi reaction with a  $^{208}$ Pb(p,d)-

<sup>207</sup>Pb experiment<sup>7</sup> carried out at the same energy with the same apparatus to check the strengths in the <sup>208</sup>Bi multiplets compared with the transition strength to the single-hole states in <sup>207</sup>Pb.

## II. EXPERIMENTAL

The experiment was carried out using the 35-MeV proton beam from the Michigan State University isochronous cyclotron. For the high-resolution experiment the reaction products were analyzed in an Enge split-pole spectrograph and the deuterons were detected in NTA and NTB  $25-\mu \rm m$  nuclear emulsions. Thin Mylar absorbers were placed in front of the emulsions to eliminate tritons. The bismuth target used was 100  $\mu \rm g/cm^2$  thick evaporated onto a  $20-\mu \rm g/cm^2$  carbon backing.

Before making exposures the total resolution was optimized by passing first the elastically scattered protons and then the deuterons from the ground state of the  $^{208}$ Pb $(p,d)^{207}$ Pb reaction, into the "speculator" system8 in the focal plane of the magnet. The basic method matches the dispersion of the beam transport system with the spectrograph dispersion to compensate for a coherent energy spread on the target. The on-line resolution meter allowed the final optimization of the dispersion of the beam on target and of the position of the plate holder and resulted in an improvement of more than a factor of 2 in resolution. We thus achieve a total resolution of less than 5 keV full width at half maximum (FWHM) for scattering angles from 6 to 50°. Two exposures were taken at each angle, one for the p and f multiplets and the other longer exposure to obtain adequate statistics on the  $h_{9/2}$ and  $i_{13/2}$  multiplets. In order to fully exploit the high resolution it was necessary to scan the nucle-

# 35-MeV Proton Inelastic Scattering from Low-Lying States in <sup>207</sup>Pb<sup>†</sup>

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Differential cross sections for the excitation of the first four excited states in <sup>207</sup>Pb via the inelastic scattering of 35-MeV protons have been compared with microscopic-model predictions. The effects of noncentral forces in a valence orbital model have been investigated and contributions from exchange and core polarization are important. Addition of an imaginary part to the microscopic form factor was also investigated and gave the best predictions of observed transition strengths.

At present, it is widely accepted that  $^{208}$ Pb is the best "closed-shell" nucleus and that nuclei in the Pb region offer an important test of the nuclear shell model.¹ To say that  $^{208}$ Pb is a good "closed-shell" nucleus does not imply that it behaves as an inert core in nearby nuclei. The need for an effective charge² to explain  $E2 \gamma$ -transition rates in neighboring systems is evidence for the coupling of the  $^{208}$ Pb core to the valence particles or holes. Detailed information about this coupling is essential to understanding both the structure of nuclei and the reaction mechanism in this region of the Periodic Table.

This letter examines the inelastic scattering of 35-MeV protons from low-lying states in <sup>207</sup>Pb. Inelastic scattering of 20.2-MeV protons from <sup>207</sup>Pb has been studied<sup>3</sup> and calculations<sup>3-5</sup> with a simple valence hole model and only central forces could not fit the data. At 35 MeV, any compound-nuclear effects contributing at the lower energy should be negligible. Further, the interactions used here had no adjustable parameters; this allows distinct separation of contributions from knock-on exchange, complex coupling, core polarization, and central and noncentral forces.

The differential cross sections were measured using the Michigan State University cyclotron. The 6.9-mg/cm²-thick self-supporting target, isotopically enriched to 99.14% <sup>207</sup>Pb, was prepared by rolling. Particle detection and identification were accomplished using a position-sensitive proportional counter and back-up scintillator-phototube<sup>6</sup> in the focal plane of an Enge split-pole spectrometer. Comparison of the experimental elastic cross sections with the optical-model predictions using Becchetti-Greenlees<sup>7</sup> best-fit parameters gave the absolute normalization. This normalization is probably reliable to about ±5%. The first four excited states were clearly separat-

ed while the doublet at 2.6-MeV excitation energy was unresolved. The collective model deformation parameter,  $\beta_3$ , for this doublet was found to agree with a previous measurement.<sup>8</sup>

To explain the measured angular distributions for the scattering from the first  $\frac{5}{2}$ ,  $\frac{3}{2}$ ,  $\frac{13}{2}$ , and  $\frac{7}{2}$  excited states in <sup>207</sup>Pb, initially distorted-wave Born-approximation (DWBA) calculations were made which included only the valence orbits. Figure 1 compares these theoretical results with the data, the error bars indicating statistical errors where they are larger than the symbols.

The valence calculations shown in Fig. 1(a) used a central nucleon-nucleon force and an approximate treatment of knock-on exchange.9 For the direct amplitude, the projectile-target interaction was taken to be the two-body effective bound-state interaction (G matrix) derived from the Hamada-Johnston (HJ) potential. The use of a similar interaction, when the transition density was known from electron scattering experiments, has given a good description of inelastic scattering.10 In these calculations, harmonic-oscillator wave functions were used with the size parameter  $\alpha = 0.405$ fm<sup>-1</sup> which reproduces the results of elastic electron scattering on 208 Pb. The optical-model parameters used were those of Becchetti and Greenlees,7 although use of other sets gave similar results. In Fig. 1(a), only the dominant S=0, L=Jtransitions are displayed. The calculations underestimate the data.

A previous study, <sup>11</sup> using central and tensor forces for the 20-MeV data, <sup>3</sup> suggested important tensor contributions in the transition to the  $\frac{3}{2}$  state. To determine if noncentral forces could significantly improve the fits, calculations were carried out still assuming a simple valence description of <sup>207</sup>Pb and using the code DWBA70<sup>12</sup> which allows the use of noncentral forces and

# $\beta$ decay of $^{22}$ F $^{\dagger}$

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 $^{22}$  F was produced in the  $^{18}$  Q( $^{6}$  Li, 2p)  $^{22}$  F reaction using 26-MeV  $^{6}$  Li  $^{3+}$  ions. Following transfer of the target in a shuttle system, delayed  $\gamma$  and  $\beta$  rays were observed with Ge(Li) and NE102 plastic detectors. Several  $\gamma$  rays from  $^{22}$  F decay were observed, decaying with a half-life of  $4.23\pm0.04$  sec.  $\gamma$ -  $\gamma$  coincidence measurements have established that a 1900.0-keV transition terminates on a level in  $^{22}$ Ne at 5523.4 keV, contrary to a previous study. The ground state of  $^{22}$ F is of  $J^{\pi}$  =  $4^+$  (with  $3^+$  as a remote possibility) disagreeing with a previous assignment of  $5^+$ . Accurate excitation energies are presented for seven states in  $^{22}$ Ne. Experimental results compare favorably with full (2s-1d) basis shell-model calculations of the A = 22 system.

RADIOACTIVITY <sup>22</sup>F; measured  $t_{1/2}$ ,  $E_{\gamma}$ ,  $I_{\gamma}$ ,  $E_{\beta}$ ,  $\gamma$ - $\gamma$  coin.,  $\gamma$ - $\beta$  coin., deduced decay scheme, J of <sup>22</sup>F,  $\log ft$ ,  $E_{\rm levels}$ ; compared with theory.

# I. INTRODUCTION

As part of a program of investigating the properties of neutron-rich nuclei in the (2s-1d) shell using heavy-ion compound reactions, we have studied the  $\beta$  decay of the  $T_z=2$  nuclide  $^{22}F$ .

A specific interest in the present case is that the A = 22 system is the heaviest (in the sense of numbers of particles or holes outside a closed shell) in the (2s-1d) shell for which extensive fullbasis shell-model wave functions are available.2 As more particles are added beyond the 16O closed shell the dimensionality of the matrices, which must be diagonalized in a complete conventional "exact diagonalization" treatment of the shell-model problem, increases very rapidly. For A = 22 the dimensionalities exceed  $500 \times 500$ . It is clearly important to test the complete shellmodel predictions as fully as possible in the heaviest systems for which they can be made to see whether-when the wave functions become of very great complexity, with hundreds of components in

terms of their original basis—they retain the reliability that they possess for the lighter systems. Only by such studies can we assess the demands that must be placed on the truncation systems, if they are to be acceptable substitutes for the full calculations, and the degree to which persistence with full calculations of even greater complexity deeper into the shell might be justified.

Comparison of shell-model predictions with experiment for A=22 is also of special interest because these nuclei are known to be among the most highly deformed of light nuclei. The shell model<sup>2</sup> is known to predict such collective features of  $^{22}$ Ne as the large B(E2) values for the groundstate band. It is then of interest to test, in the same nuclear systems, operators which are not dominated by collective effects. The comparison of theory with experiment for  $\beta$ -decay transition rates provides just such a test.

The theoretical level scheme<sup>2</sup> for the T=1 states of A=22 on the full  $(2s-1d)^6$  basis is shown in Fig. 1 together with those levels of <sup>22</sup>Ne that

# Fermi beta decay: The masses of <sup>22</sup>Mg, <sup>26</sup>Si, <sup>30</sup>S, and <sup>34</sup>Ar<sup>†</sup>

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The Q values for (p,t) reactions leading to  $^{22}$ Mg,  $^{26}$ Si,  $^{30}$ S, and  $^{34}$ Ar have been measured, yielding mass excesses for these nuclei of  $-399\pm3$  keV,  $-713\pm3$  keV,  $-14\,062\pm3$  keV, and  $-18\,379\pm3$  keV, respectively. The results for  $^{22}$ Mg and  $^{34}$ Ar are significantly different from average values quoted in the 1971 Mass Tables. For all four nuclei, these results allow the ft values for their Fermi  $\beta$ -decay branches to be calculated with improved precision, and in the cases of A=26 and A=34 more exacting comparisons of mirror Fermi decays become possible. The measured asymmetries now are  $\Delta(26)=-1.2\pm1.6\%$  and  $\Delta(34)=0.2\pm0.6\%$ , thus removing any obvious disagreement with calculations of charge-dependent mixing.

NUCLEAR REACTIONS <sup>24</sup>Mg, <sup>28</sup>Si, <sup>32</sup>S, <sup>36</sup>Ar(p, t), E = 40.2 MeV; measured Q values. <sup>22</sup>Mg, <sup>26</sup>Si, <sup>30</sup>S, <sup>34</sup>Ar deduced masses, ft values, mirror asymmetries for A = 26, 34.

## I. INTRODUCTION

 $\beta$  decay between 0<sup>+</sup> analog states has a special significance in the study of weak interactions. Since the axial-vector interaction is forbidden, the transition strength is determined entirely by the vector coupling constant and the Fermi matrix element connecting the two states. The matrix element is quite independent of the over-all nuclear structure of the states, being sensitive only to differences between their wave functions, so if the states were exact analogs of one another the vector matrix element could be calculated without recourse to a nuclear model. Comparison of the result with experimentally measured transition strengths would then yield a value for the vector coupling constant of nuclear  $\beta$  decay.

A simple test of this procedure is made possible by the availability of accurate measurements for a number of different superallowed  $0^+ \rightarrow 0^+$  (T=1) $\beta$  transitions. Obviously, if the assumption of analog-state purity were correct, the value of the coupling constant derived from all transitions would be the same. However, although the measured values do in fact lie within a few percent of one another, the scatter in the existing data is considerably greater than can be accounted for by the quoted experimental uncertainties.1-3 The answer to why this should be so must certainly involve a complete account of the extent to which charge-dependent mixing alters the initial and final nuclear wave functions, 2 but it must also include detailed radiative corrections  $^{2-5}$  and very

likely a reassessment of much of the experimental data (see, for example, Refs. 6 and 7).

As an aid to understanding these problems, it has recently been shown that some of the effects of charge-dependent mixing can be isolated from the radiative corrections by comparing the strengths of two superallowed transitions within a single multiplet. Any difference between these strengths must be caused by charge-dependent forces, and the magnitude of the difference reflects the effect of mixing on each individual transition. This provides the only experimental check on calculations for these effects. The two multiplets for which such a comparison has been possible are A = 26 and A = 34. For the latter, an apparent asymmetry between the mirror decay branches has been noted. 6.8

The intensity of each superallowed (Fermi) transition can be expressed in terms of an ft value given by<sup>2</sup>

$$fl(1 + \delta_R) = K/(G_v^{\prime 2} | M_v |^2)$$
 (1)

with  $K=1.230\ 63\times 10^{-94}$  cgs units,  $G_v'^2=G_v^2(1+\Delta_R)$  and for transitions between T=1 states  $|M_v|^2=2(1-\delta_c)$ . Here f is the statistical rate function, t is the partial half-life for the transition,  $G_v'$  is the effective vector coupling constant,  $M_v$  is the Fermi matrix element, and the "inner" and "outer" radiative correction terms appear as  $\Delta_R$  and  $\delta_R$ , respectively. Finally,  $\delta_c$  is a correction term that modifies the Fermi matrix element as a result of Coulomb and other charge-dependent forces. If we denote by the superscript "+" the positron

# Capillary waves in a quantum liquid

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Capillary waves in a superfluid are shown to propagate with the classical form of the dispersion,  $\omega^2 \approx k^3$ , but the coefficient in the model is not given by the surface tension.

The Bose quantum liquid behaves like a classical irrotational fluid in many respects. It is of interest to see whether the surface modes of the fluid are the same as the classical ones, since it is now possible to study surface waves of rather short wavelength. I will derive here the dispersion of capillary waves in a microscopic model. Using a variational principle, we find that the dispersion has the same functional form as in the classical liquid.<sup>2</sup>

$$\omega^2 = Ak^3. \tag{1}$$

We do not get the same coefficient A as appears in the classical formula,

$$\omega^2 = (\sigma/\rho)k^3, \qquad (2)$$

where  $\sigma$  is the surface tension of the fluid and  $\rho$  is the density. However, since we only have a variational energy, we cannot assert that the capillary-wave propagation in a superfluid is nonclassical.

This derivation of quantum-mechanical capillary waves follows the method Pitaevskii used<sup>3</sup> in his calculation of the dispersion of waves on a vortex line. However, his dynamical equations must be generalized somewhat in order for the system to have a free surface. We consider an energy functional of the system of the form

$$E = \int dv \left( -\psi \frac{\hbar^2}{m} \nabla^2 \psi + V[\rho] \right), \text{ with } \rho = \psi^* \psi.$$
 (3)

In Eq. (3) V is an operator on the density function; for example,  $V[\rho] = \int d^3r_1 \, \rho(r_1) \rho(r) \, V(r_{12})$ . The usual microscopic theory assumes an interaction energy density

$$V[\rho] = \frac{1}{2}V_0\rho^2(\gamma)$$
.

with  $V_0$  repulsive. Recently, progress has been made in theoretical nuclear calculations<sup>4</sup> using a more general form of V, such as  $V[\rho] \sim \frac{1}{2} V_0 \rho^2 + \frac{1}{3} V_1 \rho^3$ . A free surface is possible with suitably chosen V of this form. Minimization of the energy yields a Schrödinger-like equation for the condensate wave function of the ground state,  $\psi_0(r)$ ,

$$-\frac{\bar{n}^2}{2m}\nabla^2\psi_0 + \frac{\delta V[\rho]}{\delta \rho}\psi_0 = \lambda \psi_0. \tag{4}$$

We assume that the excited state is given by a condensate wave function of the form

$$\psi_{\mathsf{ex}} = \psi_0 + e^{i\,\omega t} f(\bar{\mathbf{x}}) + e^{-i\,\omega t} \epsilon f^*(\bar{\mathbf{x}}). \tag{5}$$

Here the function f and the constant  $\epsilon < 1$  are as yet undetermined. It is convenient to make a change of variables as follows:

$$\eta = f + \epsilon f^*, \quad \zeta = f - \epsilon f^*.$$
(6)

The variable  $\eta$  is proportional to the density fluctuation in the excitation, and the variable  $\zeta$  is proportional to the fluctuation in the velocity potential. The random-phase-approximation (RPA) equations that Pitaevskii obtains may then be expressed as follows, setting  $\hbar^2/m = 1$ :

$$\Delta \zeta = -\frac{1}{2} \nabla^2 \zeta + \frac{\delta V}{\delta \rho} \zeta - \lambda \zeta = \omega \eta , \qquad (7a)$$

$$\Sigma \eta = -\frac{1}{2} \nabla^2 \eta + \frac{\delta V}{\delta \rho} \eta + 2 \frac{\delta V^2}{\delta \rho^2} \rho \eta - \lambda \eta = \omega \zeta.$$
 (7b)

These equations may also be obtained by linearizing the hydrodynamic equation of Gross.<sup>5</sup> He derives the pair of equations

$$\frac{\partial}{\partial t}R = -\nabla R \cdot \nabla S - \frac{1}{2}R\nabla^2 S$$
 (continuity), (8a)

$$-\frac{\partial}{\partial t}S = (\nabla S)^2 + \frac{\delta V}{\delta \rho}(R^2) - \frac{\nabla^2 R}{R} \quad \text{(Bernoulli)}. \tag{8b}$$

To make the connection between Eq. (7) and Eq. (8), we identify R with  $\psi_0 + \eta e^{i\omega t}$ , S with  $\zeta e^{i\omega t}/\psi_0$ , and linearize.

An upper bound on the energy of the excitation may be found from the Thouless variational principle, which uses a trial  $\eta_t$  and a trial  $\zeta_t$ . If linear combinations of  $\eta_t$  and  $\zeta_t$  are orthogonal to all vectors associated with zero energy, such as uniform translation or rotation, then the variational  $\omega$  will be given by the upper bound

$$\frac{\langle \eta_t \Sigma \eta_t \rangle + \langle \xi_t \Delta \xi_t \rangle}{|\langle \eta_t \xi_t \rangle|} \ge \omega. \tag{9}$$

To use this principle let us assume that the surface

# $^{207}$ Pb $(p, d)^{206}$ Pb reaction and some matrix elements of the effective interaction\*

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The cross sections for the  $^{207}\text{Pb}(p,d)^{206}\text{Pb}$  reaction have been measured relative to the cross sections for the  $^{208}\text{Pb}(p,d)^{207}\text{Pb}$  reaction to the single-neutron-hole states in  $^{207}\text{Pb}$ . The reactions were studied with 35-MeV protons and a final deuteron resolution of 5 keV (full width at half maximum). By making the same assumptions as are usually made in model calculations of  $^{206}\text{Pb}$ , the matrix elements of the effective interaction of a  $p_{1/2}$  neutron with neutrons in other orbits are derived from the experimental results.

NUCLEAR REACTIONS <sup>201</sup>Pb(p,d), E = 35 MeV; measured levels,  $\sigma(\theta)$ ; deduced S, sum rules.

## I. INTRODUCTION

There have been many calculations of the properties of <sup>206</sup>Pb.<sup>1-5</sup> Most of the calculations are based on the model (1) that the six states strongly excited in the  $^{208}$ Pb $(p, d)^{207}$ Pb reaction are single-neutronhole states and (2) that 206Pb can be described as two neutron holes distributed over these singlehole orbitals. The principal difference between the various calculations is in the choice of the two-body Hamiltonian which is used to describe the interaction between these neutron holes. In the analysis of the present experiment these same two basic assumptions are made. This procedure not only allows a more direct comparison of experiment with the predictions of the model calculations but also eliminates most of the usual uncertainties resulting from the distorted-wave Born-approximation (DWBA) analysis. By obtaining accurate single-nucleon-pickup strengths, sum-rule results can be usefully applied. Perhaps the most interesting sum rule (an energyweighted sum) determines the diagonal matrix elements of the interaction of a neutron in the  $p_{1/2}$  orbit with a neutron in one of the other singleneutron orbits. These experimentally determined matrix elements are compared directly with the predictions of Kuo and Herling.3 There is generally qualitative agreement, but there are also some interesting discrepancies.

The neutron-pickup spectroscopy on  $^{207}$ Pb has been previously studied using the (d,t) reaction by Tickle and Bardwick. For the  $2^+$  states which can be reached by mixed-l transitions, and for weak states and close doublets, the present results are often very different from the (d,t) work.

# II. EXPERIMENTAL PROCEDURE

These reactions were studied using the 35-MeV proton beam from the Michigan State University

cyclotron. The targets were isotopically enriched lead targets evaporated on  $30\text{-}\mu\text{g/cm}^2$  carbon foils. The beam on the target was monitored by recording the total charge collected in the Faraday cup and by recording the protons elastically scattered at  $90^\circ$  with a scintillation counter (NaI). These two procedures for normalizing the relative cross section at different angles gave results which agreed to a few precent. The  $90^\circ$  elastic scattering monitor was used to determine the angular distributions presented here.

In the present experiment, we are principally interested in the cross sections for the  $^{207}$ Pb(p, d)-<sup>206</sup>Pb reaction relative to the <sup>208</sup>Pb(p, d)<sup>207</sup>Pb cross sections to the single-neutron-hole states in 207 Pb. While the monitoring systems accurately determined angular distributions for each individual reaction, absolute cross sections could not be obtained as accurately as the ratio of the 207Pb- $(p,d)^{206}$ Pb cross sections to the <sup>208</sup>Pb $(p,d)^{207}$ Pb cross sections. To obtain this ratio, the (p, d)reaction on a natural Pb target, for which the relative abundance of 207Pb to 208Pb is accurately known, was studied. This procedure allows the determination of cross sections in the  $^{207}$ Pb(p, d)-<sup>206</sup>Pb reaction relative to cross sections in the <sup>208</sup>Pb(p, d)<sup>207</sup>Pb reaction to a few percent.

The absolute-cross-section normalization for both reactions is more uncertain. The absolute cross section was determined in two ways: One method was to directly measure the target thickness, the spectrometer solid angle, and the charge collected in the Faraday cup, and then calculate  $d\sigma/d\Omega$ . The second method was to measure proton elastic scattering from 30–50° and compare this with the cross section predicted by the optical model. The elastic scattering was measured with exactly the same particle-detection and beammonitoring system as was used for the study of the (p,d) reactions. The optical-model param-

# Inelastic proton scattering from <sup>138</sup>Ba and <sup>144</sup>Sm at 30 MeV\*

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Differential cross sections for elastic and inelastic scattering of 30-MeV protons by  $^{138}$ Ba and  $^{144}$ Sm have been measured with a total energy resolution for the inelastic peaks of 7–10 keV, full width at half maximum. This permitted the observation of 20 excited states in  $^{138}$ Ba and 18 excited states in  $^{144}$ Sm below  $E_x=3.4$  MeV, and the determination of excitation energies accurate to 2 keV or less for these states. Based on characteristic shapes derived from angular distributions to states of known  $J^{\pi}$ , spin-parity assignments were made for the majority of the observed states. Collective-model distorted-wave Born-approximation calculations for the observed transitions were compared to the data, and deformation parameters extracted for all states to which  $J^{\pi}$  could be assigned. The energy-level structures of these nuclei, obtained by combining the present results with information in the literature, are compared to the predictions of structure calculations based on the assumption of a closed N=82, Z=50 core.

NUCLEAR REACTIONS <sup>138</sup>Ba, <sup>144</sup>Sm(p,p')  $E_p$  = 30 MeV. Resolution 7-10 keV. Enriched targets. Measured  $\sigma(\theta)$  and  $E_x$  energies for 20 states up to  $E_x \sim 3.3$  MeV. Deduced optical-model parameters. DWBA analysis, deduced  $J^\pi$ ,  $\delta_L = \beta_L R$ , transition strengths  $G_L$  for 12 states in each nucleus. Comparison of results to shell-model calculations.

## I. INTRODUCTION

The recent interest in the "N=82" nuclei stems from a number of sources. Foremost among these is the observation that in a shell-model picture, low-lying states in these nuclei are expected to be formed predominantly from proton configurations, the neutrons having the magic number 82. Furthermore, the active proton orbits should be those of the fifth major shell, those following the magic number 50. Experimental evidence from single-proton<sup>1,2</sup> and -neutron<sup>3-5</sup> transfer reactions confirms these expectations. The limit of viability of this simple picture is indicated by the positions of the lowest-lying neutron particle-hole states found near 4 MeV in isobaric-analog-resonance experiments.  $^6$ 

Electromagnetic decay aspects of the N=82 nuclei have been studied through  $(\beta,\gamma)$ ,  $^{7,8}$   $(n,\gamma)$ ,  $^{9}$   $(n,n'\gamma)$ ,  $^{10}$   $(\alpha,xn\gamma)$ ,  $^{11}$  and  $(\gamma,\gamma')$  experiments. These experiments have been useful in assigning precise energies to the observed excited states and in limiting the possible  $J^{\pi}$  assignments of many of these states to a few values. The  $(\alpha,xn\gamma)$  studies have led to the observation of a series of isomeric  $6^+$  states in the even-even N=82 isotones.

Charged-particle inelastic scattering studies have been limited mainly to the observation of the strongly excited states. Early experiments determined the positions of the first collective  $2^+$  and  $3^-$  states. The  $(p,p'\gamma)$  and  $(d,d'\gamma)$  reactions on the even-even isotones were studied in an attempt to locate the positions of excited  $0^+$  states

in these nuclei by observing the E0 conversion. electrons emitted in the transition to the ground state. More recently, the reactions  $^{139}\text{La}(\alpha,\alpha'),^{15}$   $^{140}\text{Ce}(\alpha,\alpha'),^{15}$   $^{141}\text{Pr}(\alpha,\alpha'),^{16}$   $^{138}\text{Ba}(\alpha,\alpha),^{17}$   $^{144}\text{Sm}-(\alpha,\alpha'),^{18}$   $^{144}\text{Sm}(p,p'),^{18}$   $^{19}$  and  $^{144}\text{Sm}(^{3}\text{He},^{3}\text{He}')^{19}$  have been used to study the collective nature of the strongly excited states in these nuclei.

In this paper we present results from inelastic proton-scattering experiments performed at a bombarding energy of 30 MeV on <sup>138</sup>Ba and <sup>144</sup>Sm. Use of the high-resolution system developed by Blosser et al.20 resulted in a total energy resolution for the inelastic peaks of typically 7-10 keV full width at half maximum (FWHM). Excitation energies accurate to within 1 to 2 keV were extracted and found to be in good agreement with those obtained in  $\gamma$ -ray-decay experiments. Using empirical characteristic shapes derived from angular distributions to known states, spins and parities were assigned to the majority of the observed states. The data were also analyzed with the standard collective-model distorted-wave Born-approximation (DWBA) formalism<sup>21</sup> to extract the deformation parameters of the excited states. A preliminary report of this work has been published elsewhere.22

### II. EXPERIMENTAL PROCEDURES

Our measurements were made with 30-MeV protons from the Michigan State University sector-focused cyclotron. An Enge split-pole spectrograph was used to analyze the scattered particles. The amount of beam on target was monitored both

# Neutron-deficient isotopes <sup>64</sup>Ge and <sup>65</sup>Ge<sup>†</sup>

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The  $\beta^+$  decays of <sup>65</sup>Ge and the new isotope <sup>64</sup>Ge have been studied by  $\gamma$ -ray spectrometry with chemically separated sources. Measured half-lives are  $30.0\pm1.2$  sec for <sup>65</sup>Ge and  $63.7\pm2.5$  sec for <sup>64</sup>Ge. The decay-scheme studies have been supplemented by investigation of the <sup>64</sup>Zn( $p,n\gamma$ ) <sup>64</sup>Ga and <sup>64</sup>Zn( $p,n\gamma$ ) <sup>64</sup>Ga reactions, and the mass excess of <sup>64</sup>Ga was found to be -58.819±0.008 MeV. Excitation functions for the ( $p,n\gamma$ ) <sup>64</sup>He,  $p,n\gamma$ ) reactions induced on <sup>64</sup>Zn are presented for comparison with other work. The question of the role of <sup>64</sup>Ge in nucleosynthesis is considered.

RADIOACTIVITY <sup>64,65</sup>Ge[by <sup>64</sup>Zn( $^3$ He, xn)]; measured  $T_{1/2}$ ,  $E_{\gamma}$ ,  $I_{\gamma}$ ; deduced  $I_{\beta}$ , logft. <sup>64,65</sup>Ga, deduced levels, J,  $\pi$ ; Ge(Li) detector. NUCLEAR REACTIONS <sup>64</sup>Zn(p,  $n\gamma$ ), E=7.8-9.5 MeV; measured  $E_{\gamma}$ . <sup>64</sup>Ga deduced levels. <sup>64</sup>Zn( $^3$ He, t), E=37.6 MeV, measured Q, <sup>64</sup>Ga levels; magnetic spectrograph. <sup>64</sup>Zn-( $^3$ He, 2n), -( $^3$ He, 3n), -( $^3$ He, p2n), E=20-70 MeV, measured  $\sigma(E)$ .

## I. INTRODUCTION

Recently, Arnett, Truran, and Woosley (ATW)1 pointed out that certain elements on the high-mass side of the iron peak were synthesized in a supernova explosion when the temperature and density were such that significant amounts of <sup>4</sup>He were present. In this circumstance a quasiequilibrium is established between 4He and relatively stable  $\alpha$ -particle nuclei such as <sup>56</sup>Ni, <sup>60</sup>Zn, <sup>64</sup>Ge, etc., and the synthesis process is dominated by a sequence of  $(\alpha, \gamma)$  reactions. The final abundances of the nuclei involved then depend primarily on the  $\alpha$ -particle binding energies and on the freeze-out temperature (the temperature at which nuclear reactions effectively cease). If a nucleus is weakly bound, its abundance will be low and the chain of  $(\alpha, \gamma)$  reactions will be broken at that point.

At the time of their initial calculations,  $^{64}$ Ge was unknown and ATW used for its mass the prediction of Garvey  $et~al.^2$  With this mass,  $^{64}$ Ge turned out to be the weak link in the chain, its synthesis leading (after two  $\beta^+$  decays) to about 1% of the  $^{64}$ Zn seen in nature. However, the possibility existed that  $^{64}$ Ge was in fact more tightly bound than the theoretical prediction. In the hope of clarifying this point a number of searches for  $^{64}$ Ge were undertaken,  $^{3-5}$  but without success. This raised speculation that  $^{64}$ Ge was not detected because it had unusual properties including, perhaps, an unusual mass.

This paper reports the detection and identification of <sup>64</sup>Ge, enlarging on the description contained in a preliminary letter. <sup>6</sup> Since the publication of that letter, Davids and Goosman have also ob-

served  $^{64}Ge$  and have obtained a value for its mass with an uncertainty of  $\pm 0.25$  MeV.

The experimental work described herein comprises five sections. First, a necessary preliminary to the search for 64Ge was an investigation of the levels of the daughter 64Ga, and the reaction chosen for this purpose was  $^{64}$ Zn $(p, n\gamma)^{64}$ Ga. Second, the methods used to produce 64Ge and measure its decay are described. Third, as a check on the decay scheme (which is of key importance in the mass measurement of Davids and Goosman) the  $^{64}$ Zn( $^{3}$ He, t) $^{64}$ Ga reaction was investigated. Fourth, it was possible to obtain, concurrently with the measurements on <sup>64</sup>Ge decay, new results on the decay of 65Ge which are in disagreement with early work. Finally, the excitation functions for  $^{64}$ Zn( $^{3}$ He, 2n) $^{65}$ Ge,  $^{64}$ Zn( $^{3}$ He, 3n) $^{64}$ Ge, and  $^{64}$ Ge- $(^{3}\text{He}, p2n)^{64}\text{Ga}$  were measured, chiefly to confirm the identification of <sup>64</sup>Ge but also to compare with results obtained by Crisler et al.3 for other reactions induced by <sup>3</sup>He on <sup>64</sup>Zn.

# II. IDENTIFICATION OF 64Ge

# A. $^{64}$ Zn $(p,n\gamma)^{64}$ Ga experiments

The  $\beta^+$  decay of even-even <sup>64</sup>Ge populates states in the daughter <sup>64</sup>Ga and the identification of <sup>64</sup>Ge is based primarily on the observation of  $\gamma$  rays from the decay of <sup>64</sup>Ga levels. In order to find what  $\gamma$  rays might be expected in the <sup>64</sup>Ge decay, a preliminary experiment was carried out in which the <sup>64</sup>Zn(p,  $n\gamma$ )<sup>64</sup>Ga reaction was studied on line.

A self-supporting 1-mg cm<sup>-2</sup> target of metallic <sup>64</sup>Zn, enriched to 99.85%, was bombarded with

# Isobaric mass quartets in A = 33 nuclei\*

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The  $^{36}$ Ar( $^{3}$ He,  $^{6}$ He) $^{33}$ Ar reaction has been used to measure the mass excess of the ground state and of the first two excited states of  $^{33}$ Ar. The coefficients of the isobaric-mass-multiplet equation are calculated for the three new isobaric spin quartets which are completed by the present results.

NUCLEAR REACTIONS <sup>16</sup>O, <sup>36</sup>Ar(<sup>3</sup>He, <sup>6</sup>He), E = 70 MeV, measured Q, deduced mass excess of <sup>33</sup>Ar.

#### I. INTRODUCTION

The nucleus  $^{33}$ Ar is known to be a delayed-proton emitter and its half-life has been measured.  $^1$  However, its mass is not known and is the last remaining measurement of  $T_Z = \frac{3}{2}$ , A = 4n + 1 nuclei accessible with the ( $^3$ He,  $^6$ He) reaction. This series of nuclei has been used to test the isobaric-mass-multiplet equation (IMME), since in every case they complete  $T = \frac{3}{2}$  quartets. In this paper measurements of the mass excess of the ground state and of two excited states of  $^{33}$ Ar are described. The three completed quartets are discussed in terms of the IMME and shell-model calculations of the Coulomb displacement energies.

An accurate mass determination of <sup>33</sup>Ar is made difficult by the necessity of using a gas target since such targets are not well suited to small-angle detection in a spectrograph. An additional difficulty in the experiment was the extremely small cross section which necessitated relatively high gas pressures and hence quite large target energy-loss corrections.

# II. EXPERIMENTAL METHOD

The experiment was performed using the 70-MeV <sup>3</sup>He beam from the Michigan State University cyclotron. The method for detecting the <sup>6</sup>He particles has been described previously. <sup>2,3</sup> The main difference from the previous measurements was the use of a gas-target system, which is shown schematically in Fig. 1. The gas cell was designed for a fixed angle of 10°. The fixed angle permitted a rigid alignment of the very tight collimation required to create a very short line source at small detection angles. This arrangement prevents misalignments which could allow particles from the entrance and exit windows to enter the spectro-

graph aperture.

The entrance and exit windows consisted of  $1.96\text{-}\mathrm{mg/cm^2}\text{-}\mathrm{thick}$  Havar foils. The gas pressure in the cell ranged between 300 and 460 Torr. The isotopic enrichment of the  $^{36}\mathrm{Ar}$  gas was better than 99%. For the calibration reaction <sup>16</sup>O-(3He, 6He)13O (see Ref. 4 for the mass of 13O) the same gas cell was used with a pressure of oxygen adjusted to make the energy losses of the 6He particles in the gas cell the same for both the primary and calibration reactions. As in previous measurements the mass was determined by comparing the magnetic fields required to put the <sup>6</sup>He particles from the primary and calibration reaction at the same position on the focal plane of the spectrograph. In Fig. 2 an example of the spectra obtained is displayed. The over-all energy resolution was about 150 keV. The effect of the target energy loss on the energy resolution was of the same magnitude as the broadening of the image on the focal plane due to the extended line source. Other sources of broadening are negligible compared to these two effects. The determination of the true angle of the emission of the reaction products is important even at forward angles like  $10^{\circ}$ because of large kinematic differences between the  $^{16}O(^{3}He, ^{6}He)^{13}O$  and  $^{36}Ar(^{3}He, ^{6}He)^{33}Ar$  reactions. The angle was measured by filling the gas cell with an oxygen and helium mixture and comparing the energy of the elastically scattered <sup>3</sup>He particles from these nuclei. This established the true angle to be  $10.0 \pm 0.1^{\circ}$ .

The main source of the experimental errors was the target energy-loss determination. Since the cross section of the  $^{36}Ar(^{3}\text{He},\,^{6}\text{He})^{33}Ar(g.s.)$  was measured to be very small  $(0.13\pm0.02~\mu\text{b/sr}$  at  $10^{\circ}$ ), relatively high gas pressures were used in order to get reasonable counting rates. This re-

# A = 9 isospin quartet\*

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New measurements of the mass excess of the lowest  $T=\frac{3}{2}$  levels in  ${}^9\mathrm{B}$  and  ${}^9\mathrm{B}$  give values of  $25.7406\pm0.0017$  and  $27.0711\pm0.0023$  MeV, respectively, and indicate a definite cubic dependence for the masses of the A=9 quartet which includes  ${}^9\mathrm{Li}$  and  ${}^9\mathrm{C}$  in their ground states. A value of  $7.6\pm1.7$  keV is obtained for the coefficient of the  $T_z^3$  term which now clearly exceeds the current theoretical estimates. Precise values of excitation energy of levels in  ${}^{10}\mathrm{B}$  and  ${}^{11}\mathrm{B}$  have been obtained as part of these measurements.

NUCLEAR REACTIONS <sup>11</sup>B(p, t), (p, <sup>3</sup>He), E = 42 MeV; measured  $E_x$  of  $T = \frac{3}{2}$  levels of <sup>9</sup>Be and <sup>9</sup>B. <sup>10,11</sup>B(p, p') E = 35 MeV; measured  $E_x$  of <sup>10,11</sup>B levels; deduced coefficient of multiplet mass equation for A = 9.

### INTRODUCTION

Considerable effort has been devoted to testing the symmetry relation embodied in the mass equation for isobaric-analog nuclear states. In 1957, Wigner<sup>I</sup> showed that if one assumes isospin to be a good quantum number, the mass difference between members of a *T* multiplet is a second-degree polynomial, i.e.,

$$M(T_z) = a + bT_z + cT_z^{2}. (1)$$

This relationship has proved remarkably accurate for data that now include 15  $T=\frac{3}{2}$  quartets of nuclear states. In only one of these quartets, the lowest  $T=\frac{3}{2}$  A=9 system, does it appear that a deviation of some significance has been observed. The addition of a  $dT_z$  term to Eq. (1) above with  $d=8.0\pm3.7$  keV was required to fit the experimental data.<sup>2</sup>

Deviations from Eq. (1) would not be the result of inequalities of nuclear forces, i.e., the breaking of charge symmetry, but as pointed out in Ref. 1, would be mainly produced by the electrostatic interaction in the nucleus. Various aspects of Eq. (1), also known as the isobaric multiplet mass equation (IMME) have been discussed in detail by Garvey, <sup>3</sup> including mechanisms which generate higher-order terms.

There has been a number of calculations of the possible contributions to this d coefficient. Two recent papers, one by Hardy, Loiseaux, Cerny, and Garvey<sup>4</sup> and the other by Bertsch and Kahana, <sup>5</sup> which were both aimed at this d-term evaluation, conclude with the suggestion that the most fruitful avenue may lie in repeating some of the experimental work to ascertain the cubic dependence with better accuracy.

For A = 9 the value of d for a cubic IMME is

given by

$$d = \frac{1}{6} ({}^{9}\text{Li} - {}^{9}\text{C}) - \frac{1}{2} ({}^{9}\text{Be} * - {}^{9}\text{B*}), \tag{2}$$

where the symbols stand for the mass excess of corresponding  $T=\frac{3}{2}$  states. Since all these states have approximately the same uncertainty in their masses of about 5 keV, <sup>2</sup> Eq. (2) shows that the current experimental uncertainties in <sup>9</sup>Be\* and <sup>9</sup>B\*  $T=\frac{3}{2}$  states are 3 times more significant than those of <sup>9</sup>Li and <sup>9</sup>C in the determination of d.

# EXPERIMENTAL PROCEDURE AND RESULTS

To reach the lowest  $T = \frac{3}{2}$  level of <sup>9</sup>Be, a thin (~30  $\mu g/c\,m^2)$  target of  $^{11}B$  evaporated onto a 30- $\mu g/cm^2$  carbon backing was bombarded with 42-MeV protons from the Michigan State University cyclotron. The <sup>3</sup>He particles from the (p, <sup>3</sup>He) reactions were analyzed at a laboratory angle of  $8^{\circ}$  in an Enge split-pole spectrograph. The detection system consisted of a 12-cm-long current division wire proportional counter in the focal plane of the spectrograph followed by a plastic scintillator used for time-of-flight particle identification.6 The method for obtaining a highly precise value for the mass excess of the  ${}^{9}\text{Be }T=\frac{3}{2}$ level was to compare the energy of the 3He in that reaction to those from the  $^{12}C(p, ^3He)^{10}B$  reaction leading to the  $2^{+}$   $^{10}$ B level at  $5.166 \pm 4 \text{ keV}$   $^{7}$  for which there is a strong peak in the 3He spectrum. The energy of the <sup>3</sup>He group of interest for <sup>9</sup>Be\* is only 125 keV greater than that corresponding to the calibration peak in 10B, and the target masses, i.e., 11 and 12, are similar, and therefore a highly accurate comparison insensitive to beam energy and laboratory angle can be expected. Since the error of 4 keV is far too large for the present purpose, the first measurements involved

# $T = \frac{3}{2}$ states in mass-11 nuclei\*

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A 740-keV-wide peak in the  $^6$ He spectrum from the bombardment of  $^{14}$ N by 70-MeV  $^3$ He particles is identified as the mirror of the  $\frac{1}{2}^-$  first excited state of  $^{11}$ Be. The Q value for the  $^{14}$ N( $^3$ He,  $^6$ He) $^{11}$ N reaction to this state is  $-25.03\pm0.10$  MeV, which corresponds to a mass excess of  $25.23\pm0.10$  MeV. The analogs of this state in  $^{11}$ C and  $^{11}$ B were observed with the  $^{13}$ C(p,  $^3$ He) reactions, and the excitation energies and widths found to agree well with the previous measurements.

NUCLEAR REACTION <sup>14</sup>N( $^3$ He,  $^6$ He), E=70 MeV measured Q, deduced mass excess of  $^{11}$ N.

## I. INTRODUCTION

The nucleus <sup>11</sup>N, which is the mirror nucleus to <sup>11</sup>Be, is predicted to be several MeV unbound by Coulomb energy systematics and the Garvey-Kelson mass relations. The ground-state spin of <sup>11</sup>Be is  $\frac{1}{2}$ , and an isobaric mass quartet based on this state would be an extremely interesting test of the shell-model description, which includes strong  $2s_{1/2}$  particle strength even though the nucleus is located well within the 1p shell. However, the mirror of this level in <sup>11</sup>N would have a very large width because it can decay very rapidly by an l=0 proton to <sup>10</sup>C in its ground state. It is also very difficult to form such a state with, for example,

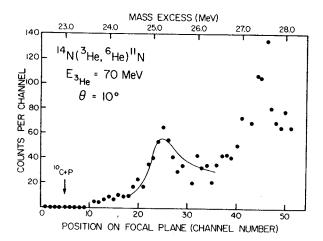


FIG. 1. A composite spectrum from several runs on the  $^{14}\rm{N}(^3\rm{He},~^6\rm{He})^{11}\rm{N}$  reaction at 70 MeV and 10° (lab). The solid curve is a theoretical estimate of the shape of the  $^{11}\rm{N}$  peak and is discussed in the text.

the  $^{14}\rm{N}(^3\rm{He}, ^6\rm{He})^{11}\rm{N}$  reaction since in this case the required  $2s_{1/2}$  particle strength is not present in the target.

The first excited state of 11Be has spin ½ (Ref. 5) and lies at  $E_x = 0.320$  MeV. The analogs of this state are known in 11B and 11C, and the decay of its mirror state in 11N would at least be hindered by the requirement of an l=1 proton decay. In this paper a peak in the spectrum of  $^{14}\mathrm{N}(^{3}\mathrm{He},\,^{6}\mathrm{He})^{11}\mathrm{N}$  is shown to have a width consistent with l=1 decay and an energy in very good agreement with the predictions of the isobaric multiplet mass equation based on the three  $T = \frac{3}{2}, \frac{1}{2}$  levels in the A = 11nuclei. In addition the cross section for 14N- $(^{3}\mathrm{He}, ^{6}\mathrm{He})^{11}\mathrm{N}$  has a typical value for p-shell target nuclei. The  $T = \frac{3}{2}$  states in <sup>11</sup>B and <sup>11</sup>C were also studied using the  $^{13}{
m C}(p,\,^3{
m He})$  and  $^{13}{
m C}(p,\,t)$  reactions in order to check the parameters of the known  $T = \frac{3}{2}$  levels.

# II. EXPERIMENT AND RESULTS

## A. Search for states in 11N

The <sup>14</sup>N(<sup>3</sup>He, <sup>6</sup>He)<sup>11</sup>N reaction was studied at a beam energy of 70 MeV and laboratory angles of 6, 10, and 13° using a time-of-flight plus magnetic analysis combination which has been previously described.<sup>6</sup> The 10° data were the most satisfactory since they were obtained with a fixed angle gas target arrangement which was designed especially for use in a spectrograph.<sup>7</sup> The 6° data were taken with a 1-mg/cm² melamine foil, the uniformity and thickness of which changed during bombardment. The 13° data were taken with a conventional gas target arrangement. The 6 and

# Configuration mixing of two-quasiparticle states in even-even deformed nuclei\*

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Evidence of configuration mixing of higher-K bands in deformed even nuclei is surveyed. A general formulation for configuration mixing due to a two-body neutron-proton force is developed. A fit to the energy splittings of Gallagher-Moszkowski pairs in odd-odd nuclei is made to obtain an effective Gaussian central force except for the undetermined Wigner component. With this force, off-diagonal band-mixing matrix elements are calculated for various configurations in  $^{176}$ Hf,  $^{178}$ Hf, and  $^{174}$ Yb. By solving BCS equations, the relevant occupation amplitudes are calculated. The effective n-p Wigner force component is fixed to give best over-all agreement to experimental band-mixing information. The resulting force is compared with the Jones, Onishi, Hess, and Sheline central force for deformed nuclei.

NUCLEAR STRUCTURE  $^{174,176,178,180}$ Hf,  $^{174}$ Yb; calculated configuration mixing higher K bands, fit energy splittings, Gallagher-Moszkowski pairs, n-p force deduced.

# I. INTRODUCTION

Among certain classes of two-quasiparticle states of deformed nuclei configuration mixing has been extensively treated, while for other classes there has been almost no attention to this problem.

On the one hand, the excited bands of  $K^{\pi}=0^+$ ,  $1^+$ ,  $2^+$ ,  $0^-$ ,  $1^-$ ,  $2^-$ , and  $3^-$  in even-even nuclei have been extensively treated microscopically. These treatments are usually carried out with some simple separable interactions (quadrupole-quadrupole, octupole-octupole, spin-quadrupole, or surface  $\delta$  interaction). Some bands in the systems treated may become "collective" and consist of a linear combination of many two-quasiparticle basis states.

On the other hand, there has been little theoretical attention to the question of configuration mixing of higher-K bands than those mentioned above, and the general question of the effective

nucleon-nucleon force appropriate in this context is quite open.

# II. EXPERIMENTAL EVIDENCE

In the past several years interesting measurements have been made concerning band mixing of two-quasiparticle states in even-even nuclei. The even-even nuclei in the region around <sup>178</sup>Hf are interesting because of their prolific isomerism, associated with the availability of only large- $\Omega$  Nilsson orbitals near the Fermi energy: For protons the orbitals involved are  $\frac{7}{2}$ [404],  $\frac{9}{2}$ [514], and  $\frac{5}{2}$ [402], and for neutrons they are  $\frac{5}{2}$ [512],  $\frac{7}{2}$ [514], and  $\frac{9}{2}$ [624]. Thus, relatively low-lying  $K^{\pi}=6^+$ , 8-, and 7- states can be formed either as two-quasiproton or two-quasineutron states.

Khoo et al. have carried out impressive measurements of excited bands in  $^{176}$ Hf. Their analysis shows that the  $K^{\pi}=6^{+}$  bands at 1333.1 and 1761.5 keV are highly mixed between two-quasi-

# SEARCH FOR A $\gamma$ -BRANCH FROM SHAPE ISOMERS IN $^{236}$ U AND $^{238}$ Nn

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Abstract: A search for  $\gamma$ -decay from the shape isomer in  $^{236}$ U ( $t_{\frac{1}{2}}=130$  ns) and in  $^{238}$ Np (hypothetical) has been performed in the reactions  $^{235}$ U(d, p) $^{236}$ U and  $^{238}$ U(p, n) $^{238}$ Np by the use of a pulsed beam. In both cases there are possible candidates, but no definite  $\gamma$ -decay has been established. The experiments yield limits on production cross sections, which are higher than the theoretical estimates.

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NUCLEAR REACTIONS <sup>235</sup>U(d, p), E=11 MeV, <sup>238</sup>U(p, n), E=8 MeV; measured  $\sigma$  (delayed  $\gamma$ ),  $T_{\frac{1}{2}}=130$  ns, 2  $\mu$ s  $< T_{\frac{1}{2}}< 20$  ms. Deduced limits on  $\sigma$  for delayed  $\gamma$  from shape isomer <sup>236</sup>mU and <sup>238</sup>mNp.

## 1. Introduction

An important contribution to the investigations  $^{1}$ ) of shape isomers in heavy elements – secondary minima in the fission barrier – would be the observation of decay modes other than fission. If one assumes that the fission isomers are indeed metastable states in a secondary minimum, then it is of interest to know the degree to which the isomer decays by penetration through the inner barrier followed by  $\gamma$ -decay to the ground state.

Such a  $\gamma$ -branch might be expected in nuclei where the inner barrier is low enough relative to the outer barrier to permit significant branching, yet still sufficiently high to cause a measurable half-life for the isomer. The liquid drop fission barrier moves towards larger deformation when  $Z^2/A$  is decreased, whereas the decreased level density which gives rise to the secondary minimum occurs at an almost constant deformation. This means that the inner barrier is expected to be low relative to the outer one at the low-Z end of the island of isomerism,  $92 \le Z \le 97$ .

Estimates of the partial  $\gamma$ -half-life based on systematics of barrier parameters are reviewed in sect. 2. In sect. 3 we describe the two experiments: One ( $^{236m}$ U) in which fission decay has been observed and the total half-life is known, and where it is expected that the  $\gamma$ -branch constitutes  $\approx 80\%$  of the decay; and the other ( $^{238}$ Np) where no fission has been observed to a very low limit and where the only depopulation of a hypothetical isomeric state is expected to be  $\gamma$ -decay.

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# NUCLEAR SPECTROSCOPIC STUDIES OF 252Es

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Abstract: The decay scheme of  $^{252}$ Es has been investigated with high-resolution semiconductor detectors in conjunction with coincidence techniques. The half-life of  $^{252}$ Es was measured by following the decay of the  $\alpha$ -count rate associated with  $^{252}$ Es decay and was found to be  $350\pm50$  d. The electron capture and  $\alpha$ -decay branchings were measured to be  $(22\pm2)\%$  and  $(78\pm6)\%$ , respectively. The EC decay almost entirely populates a level at 969.8 keV in  $^{252}$ Cf with a log ft value of 8.9. This state has been identified as the two-neutron state  $\{n[613]\frac{\pi}{2}^2; n[620]\frac{1}{2}^4\}^3$ . A  $K^{\pi} = 2^-$  band has been identified at 830.8 keV. A rotational band built on an 804.8 keV level has been interpreted as the  $\gamma$ -vibrational band  $(K^{\pi} = 2^+)$ . On the basis of the observed log ft value, the ground state of  $^{252}$ Es has been given an assignment of  $\{n[613]\frac{\pi}{2}^2; p[521]\frac{\pi}{2}^-\}^5$ . The favored  $\alpha$ -transition of  $^{252}$ Es has been found to populate a level at 590 keV in  $^{248}$ Bk. The ground state of  $^{248}$ Bk has been given a spin-parity assignment of  $6^+$  with the two-quasiparticle configuration  $\{n[734]\frac{\pi}{2}^2; p[521]\frac{\pi}{2}^-\}$ .

RADIOACTIVITY <sup>252</sup>Es [from <sup>252</sup>Cf(d, 2n), <sup>249</sup>Bk( $\alpha$ , n)]; measured  $T_{\frac{1}{2}}$ ,  $E_{\alpha}$ ,  $I_{\alpha}$ ,  $E_{\gamma}$ ,  $I_{\gamma}$ ,  $I_{ce}$ ,  $\gamma\gamma$ -,  $\gamma$ ce-,  $\alpha\gamma$ -coin,  $\alpha$ /EC ratio; deduced log ft. <sup>252</sup>Cf and <sup>248</sup>Bk deduced levels, J,  $\pi$ ,  $\gamma$ -multipolarity.

# 1. Introduction

The nuclide  $^{252}$ Es decays predominantly by  $\alpha$ -particle emission and populates levels in doubly odd  $^{248}$ Bk. Investigation of energy levels in heavy doubly odd nuclei is important because these data are needed for a better understanding of the neutron-proton interaction in the nucleus. The  $\alpha$ -decays of only two doubly odd nuclides ( $^{242m}$ Am and  $^{254}$ Es) have been studied  $^{1,2}$ ) in detail. These studies have resulted in the identification and characterization of several rotational bands in  $^{238}$ Np and  $^{250}$ Bk.

The  $\alpha$ -decay of  $^{252}$ Es was first investigated by McHarris  $^3$ ) who measured  $\alpha$ -singles and  $\alpha$ - $\gamma$  coincidence spectra. Because of the availability of only a small amount of  $^{252}$ Es and the presence of a large quantity of  $^{254}$ Es in the sample sufficient information could not be obtained to formulate a complete decay scheme. A partial decay scheme was postulated and a spin-parity assignment of  $7^+$  was made to the  $^{252}$ Es

<sup>†</sup> Work supported in part by the USAEC and the US National Science Foundation.

# SHELL MODEL CALCULATIONS FOR MASSES 27, 28 AND 29: SPECIFIC APPLICATION TO <sup>27,28</sup>Mg AND <sup>28,29</sup>Al

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## Received 25 January 1973

Abstract: A shell model calculation has been performed in a truncated  $1d_{\frac{1}{2}}$ ,  $2s_{\frac{1}{2}}$ ,  $1d_{\frac{1}{2}}$  configuration space with a modified surface-delta-interaction Hamiltonian for the T=1,  $\frac{3}{2}$  and 2 nuclei of A=27-29. The parameters of the model Hamiltonian were obtained from a fit to experimental data from T=0 and  $T=\frac{1}{2}$  nuclei of the same region. Excitation energies, single-nucleon spectroscopic factors, electric quadrupole moments and transition strengths and magnetic dipole moments and transition strengths were calculated. Decay properties of the T=2 and T=1 multiplets in A=28 and the  $\Delta T=1$ , M1 transitions from  $T=\frac{3}{2}$  levels in A=27 and 29 nuclei have been studied in particular. The results indicate that the present shell model is able to describe quantitatively the nuclear structure around mass 28 in a unified approach in a fashion similar to the excellent descriptions available in this same theoretical framework for lighter and heavier mass regions of the sd shell.

# 1. Introduction

Nuclei around mass 28, where the nuclear shape changes from prolate to oblate  $^1$ ), have been the object of many extensive theoretical and experimental studies, of which the results are compiled in ref.  $^2$ ). Discussions in terms of collective models are mostly concentrated on a few aspects of particular nuclei. The collective models frequently account very well for certain characteristics  $^{4, 5, 9, 10}$ ) but they all fail, variously, to explain at the same time many other significant features in A=27-29 nuclei. Of the four nuclei considered here, only the structure of  $^{27}$ Mg is reasonably well understood in terms of the single-particle Nilsson model  $^{3-9}$ ), and even here deformation parameters of different signs are required to explain the transition rates  $^{4, 5}$ ) and the spectroscopic factors  $^9$ ). No significant evidence was been found so far to place  $^{28}$ Al [ref.  $^{11}$ )] and  $^{29}$ Al [refs.  $^{12-15}$ )] in a simple rotational picture, while an intermediate-coupling version of the SU(3) model  $^{16}$ ) reproduces only some of the characteristics of the  $^{28}$ Mg energy spectrum.

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# FROM <sup>40</sup>Ca, <sup>120</sup>Sn, AND <sup>208</sup>Pb<sup>†</sup>

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Abstract: Theoretical (e, e') form factors and (p, p') differential cross sections for the first  $3^-$  and  $5^-$  excitations in  $^{40}$ Ca, the first  $3^-$  excitation in  $^{208}$ Pb, and the first  $2^+$  and  $3^-$  excitations in  $^{120}$ Sn are presented and compared with experiment. Results are also presented which test the hypothesis that the proton and neutron transition densities for these transitions are related by the condition  $\rho_a = (N/Z)\rho_p$ . A simple modified Born approximation has been used in the electron scattering calculations. The long range part of the Kallio-Kolltveit potential has been used for the projectile-target interaction in the proton scattering calculations and "knock-on" exchange contributions have been included approximately.

## 1. Introduction

Inelastic electron scattering and inelastic proton scattering are well known tools for the study of nuclei. In the inelastic excitation of a nuclear collective state, an electron is essentially scattered only by the target protons <sup>1</sup>). On the other hand, a proton with energy < 100 MeV interacts 2-3 times more strongly with the target neutrons than it does with the target protons in bringing about the same transition [refs. <sup>2-4</sup>)]. As a result, comparison of electron scattering and proton scattering allows separate discussion of the proton and neutron transition densities, i.e. those functions which describe the motion of the target nucleons during the transition.

The electron-nucleus interaction is electromagnetic in origin and well understood in principle. In addition, an electron is not absorbed appreciably during the scattering process. These features make possible an accurate determination of the proton transition density directly from the experimental data, provided it extends over a sufficient range of momentum transfer. There are some theoretical uncertainties in the interpretation of the (p, p') reaction and a proton is absorbed as it is scattered, so the information gained is not so precise as in the case of electron scattering.

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# THE ERROR OPERATOR AND ITS EIGENVECTORS IN ELECTRON SCATTERING

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## Received 22 October 1973

Abstract: We make a formal analysis of the error in the nuclear charge density when determined from electron scattering. The error statement is made by introducing error functions describing asymptotic behaviour of the density and the form factor. It is possible to make a complete error statement using the above functions, so that there is no need to use a truncated basis. Consequently, our analysis is basis independent. Because we have introduced an error operator which has one part analogous to a potential (because it can be written simply in coordinate space) and another part which might be made analogous to the kinetic energy (because it is simply written in momentum space) we have made the analogy of the error operator to the Hamiltonian. We find that results of our previous paper remain valid and the conclusions are unaltered but the present procedure lends greater mathematical rigor.

# 1. Introduction

In the previous paper 1) (which will be called I) we demonstrated that electron scattering could be analyzed equally well using one out of a wide class of sets of functions used to expand the density.

Here we attempt to show this independence in a more rigorous way.

We want to emphasize that we do not claim "model independence". We only claim that the model does not need to be specified by giving a set of functions to be used in the fitting. Instead, the model can be specified in the form of error expectations for the deviation of the form factor and charge distribution from zero in the large q and r regions respectively.

We show below that a statement of these error expectations leads to a unique charge density and expected error which is independent of the choice of expansion functions. The results are not very different numerically from those we obtained before. The general conclusions of I about the expected error of  $\rho(r)$  remain valid.

# 2. Mathematical formulation

We will again restrict ourselves to the situation where Born approximation is valid. We introduce a linear vector space with elements  $|x\rangle$ . One of the vectors in this space,  $|\rho\rangle$ , corresponds to the physically measured quantity. In the coordinate space representation,  $\langle r|\rho\rangle = \rho(r)$ , this vector is just the radial charge distribution

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# RECOIL CORRECTIONS FOR SINGLE-NUCLEON TRANSFER REACTIONS<sup>†</sup>

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# Received 11 January 1974

Abstract: A formalism is developed for approximately treating the recoil of the projectile in single-nucleon transfer reactions. The "no-recoil" term is evaluated exactly by rotating the nuclear matrix element to a coordinate system where the lighter projectile lies along the z-axis relative to the target nucleus. Recoil terms are constructed by expanding the "recoil phase factor" in terms of spherical harmonic and Bessel functions, and they are again evaluated exactly. Numerical results are then obtained for some representative light-ion reactions of high incident energies. For these reactions, we find that although the first-order recoil term may be very large, higher order terms are correspondingly less important, and convergence is always easily obtained. However, absolute magnitudes of cross sections at forward angles predicted by the present theory and by the local energy approximation differ by up to 50% for the reactions we consider.

### 1. Introduction

Recoil corrections to single-nucleon transfer reactions have been considered many times in the past, both by exact <sup>1-4</sup>) and approximate <sup>5,6</sup>) means. Unfortunately, the exact finite-range calculations have been found to be quite costly and time consuming, and so there still remains an active interest in finding alternate ways of suitably treating recoil. Such a procedure is developed in the present work.

One limitation of the two approximate methods that have so far been considered for recoil <sup>5,6</sup>) is that they both impose some rather severe restrictions on the wavefunction of the transferred particle bound in the target nucleus. Dodd and Greider <sup>5</sup>) approximate this wavefunction with harmonic oscillator orbitals, which are realistic only for deeply bound wavefunctions, while Nagarajan <sup>6</sup>), on the other hand, considers a spherical Hankel function valid only for weakly bound particles. In the present work, we formulate a theory valid for wavefunctions of arbitrary numerical shape. Although the resulting form factors are not analytic, we will show that they can be rapidly computed by numerical means and readily inserted into standard zerorange DWBA codes.

In sect. 2, we present our method for effectively computing the recoil terms. Then, in sect. 3, we specialize the formalism to light-ion projectiles where the transferred

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# **Nuclear Response Function**

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A fast numerical method is described for calculating nuclear excitation properties with  $\delta$ -type interactions. The method is applied to  $^{208}\mathrm{Pb}$ , and the following properties of the nucleus are found: (i) The quadrupole strength has two main pieces, one a low state and one identifiable as the giant quadrupole; (ii) excitations with L > 2 do not seem to have high-energy collective parts; (iii) the giant dipole L=1, T=1 is too low unless the interaction has a strong momentum dependence.

There has been much interest recently  $^{1-3}$  in Hartree-Fock calculations of nuclei using the parametrization of the nucleon interaction in terms of momentum and density-dependent  $\delta$  functions, called Skyrme interactions.  $^4$  If the Hartree-Fock description of a ground state is reasonable, the random-

# Excitation of Giant Resonances by Inelastic <sup>3</sup>He Scattering\*

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Inelastic  $^3$ He scattering at 71 MeV on twelve nuclei ranging from  $^{27}$ Al to  $^{209}$ Bi shows an enhancement of the continuum very similar to that observed in electron and proton scattering. The effect is ascribed to a giant multipole state. The strength of the excitation indicates an E2 character for the state.

Inelastic scattering of <sup>3</sup>He particles favors states of collective character in the final nucleus. and therefore this process should excite strongly the regions of the continuum in which the various multipole strengths are located. Recent work, however, is conflicting concerning the excitation of the proposed E2 giant resonance by inelastic scattering of <sup>3</sup>He particles. <sup>1,2</sup> Since such a state could exhaust a large fraction of the energyweighted sum rule<sup>3</sup> (EWSR) in a relatively narrow region of excitation energy (3-4 MeV), it should appear strongly in <sup>8</sup>He spectra. The energy of the state is predicted to lie 2 MeV below the giant dipole resonance (GDR), and this appears to be well corroborated by inelastic electron and proton scattering.4-9 However, the effects observed in these experiments can also be explained by the assumptions of an E0 giant resonance. 10

Previous studies 1.11 of 3He and 4He scattering from Pb and Au at several forward angles indicate some enhancement in the region 2 MeV below the GDR, whereas at 41 MeV no enhancement of the continuum was observed in 3He scattering from 24Mg, 26Mg, 50Cr, 60Ni, and 90Zr.2 Broad enhancements of the continuum at high excitation energies present special experimental problems. For example, slit scattering and nonlinearities or dead regions of the detection apparatus can easily produce or, alternatively, obscure such effects. The present experiment was undertaken to determine whether the effect observed in 3He scattering is real, and, if so, to attempt to test the E2 character of the excitation.

The spectra of <sup>3</sup>He particles scattered from nuclei from <sup>27</sup>Al to <sup>209</sup>Bi were detected at forward angles. The data shown in Fig. 1 were taken with a silicon detector telescope. Similar spectra were obtained with a current-division wire proportional counter on the focal plane of a spectrograph. A plastic scintillator behind the wire counter gave total energy and time-of-flight information. This setup gave very clean spectra which were essentially identical to the data shown

in Fig. 1 except that a smaller range of excitation energy was covered. Target thicknesses were kept relatively large (~2 mg/cm²) to minimize the relative yield from light contaminants in the target. The energy of the beam was 71 MeV, and the energy resolution was typically 200 keV. In Fig. 1 spectra from seven nuclei ranging from <sup>27</sup>Al to <sup>209</sup>Bi are shown at a lab angle of 20°. No subtractions or corrections have been applied to the data. Similar results were obtained for <sup>54</sup>Fe, <sup>120</sup>Sn, and <sup>197</sup>Au, and, in fact, no targets which were bombarded failed to display an enhancement of the continuum like that shown in Fig. 1.

There is a very strong yield to excitation energies 2-3 MeV below the GDR, the position of which is shown by an arrow in Fig. 1, and the shape of the peak is asymmetric with the lower-energy edge being much sharper and more well defined. The width of the observed structure is considerably wider than that expected from photonuclear reaction for the GDR. However, in the lighter nuclei ( $A \le 58$ ) the spectra exhibit a considerable amount of structure just as they do in photonuclear reactions. The differential cross sections are very forward peaked with a rate of falloff similar to direct excitation of known collective states at the same beam energy.

The proposed E2 state and the GDR cannot be resolved from each other because the width of the two states exceeds their separation. Hence, the strength and position of the E2 state can only be estimated under various assumptions and then assigned an appropriate error. One procedure which gave reasonable results on the heavier nuclei consisted of the following steps: (1) A flat background was subtracted using points well below and above the GDR; (2) the largest possible GDR contribution was subtracted using the known width and position from photonuclear work; (3) the resulting peak was checked to see if it moved correctly kinematically. It displayed, in fact, a smooth symmetric shape about 3 MeV wide. The flat background, the assumed GDR, the

# Experimental Demonstration of Backbending Behavior from a Band Crossing in 154Gd†

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The members of the ground and  $\beta$  bands in <sup>154</sup>Gd have been identified to spin 18. Plots of  $2g/\hbar^2$  versus  $\omega^2$  yield backbending curves for both the  $\beta$  and yrast states. It is shown that the backbending in the yrast sequence results from a band crossing and it is speculated that the twin backbending may arise from the intersection of the ground and  $\beta$  bands by a third "intersecting band."

At high spins the rotational bands of a number of nuclei exhibit a phenomenon commonly referred to as backbending—a name derived from the characteristic S-shaped plots of the moment of inertia,  $29/\hbar^2$ , versus the square of the nuclear rotational frequency,  $(\hbar\omega)^2$ . This anomalous behavior has been attributed to the Mottelson-Valatin4 effect, which is a phase transition from the normal superfluid state to one in which the nucleon pairs have been coherently broken by the Coriolis force. It is also possible to explain the backbending effect in terms of a band which intersects the ground band to become the yrast band. 5,6 The question then reduces to one of the intrinsic character of the intersecting band: It might be the Mottelson-Valatin4-type unpaired band, the decoupled band of Stephens and Simon, or perhaps even a quasiparticle or vibrational band. However, there has been no experimental evidence that explicitly demonstrates the band-crossing feature. We shall show here a case in which backbending in the yrast band can be directly attributed to band crossing. It would, however, be premature to generalize from this particular case that all other cases of backbending can be similarly explained.

The levels of  $^{154}$ Gd have been studied by means of the reaction  $^{154}$ Sm( $\alpha$ , 4n) $^{154}$ Gd using  $\alpha$  beams from the Michigan State University sector-focused cyclotron. Measurements of the angular distri-

butions of the  $\gamma$  rays and  $\gamma$ - $\gamma$  coincidence experiments were performed with 48-MeV  $\alpha$  beams, while excitation function measurements were made with beams of 41, 45, 48, and 50 MeV. Supplementary singles data from the reaction  $^{152}{\rm Sm}(\alpha,2n)^{154}{\rm Gd}$  with 24-MeV  $\alpha$ 's were also accumulated.

Two rotational bands—ostensibly the ground and  $\beta$  bands—have been identified and are shown in Fig. 1. The spin assignments were based on the angular-distribution and excitation-function data in the usual manner and also on the relative intensities of the  $\gamma$  rays in the  $(\alpha, 4n)$  and  $(\alpha, 2n)$ reactions. The assignments for the levels up to spin 16 agree with those previously reported. 7,8. Each of the two spin-18 members, which have not been observed before, decays to both bands and there is an ambiguity regarding their assignment to the respective bands; we have chosen to group together in Fig. 1 the yrast and the yrare levels. [An yrare level is defined here as the first excited state above the lowest (yrast) state of the same spin.

The interband-to-intraband B(E2) ratio, B(E2, I'+I-2)/B(E2, I'+(I-2)'), given in Table I and in parentheses in Fig. 1, increases dramatically from  $<10^{-3}$  to 1.4 as I' increases from 12 to 18; in addition B(E2, 18-16')/B(E2, 18-16)=B(E2, 18'+16)/B(E2, 18'-16'), within experimental errors. These features can be explained by con-

# Backbending and Forking in the Yrast States of Even Os Isotopes\*

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The yrast levels have been determined to spin 16 and 18 in  $^{182,184,186}$ Os and to spin 12 in  $^{188}$ Os from  $\gamma$ -ray excitation functions, anisotropies, and coincidence data from the reactions  $^{182,184,186}$ W( $\alpha$ , 4n) and  $^{186}$ W( $\alpha$ , 2n). A fork is observed in the  $^{186}$ Os yrast band at spin 12. Above spin 12 the  $\theta$ -versus- $\omega^2$  curves for  $^{182,184,186}$ Os display "backbending" behavior. Such behavior has not been reported for any other nuclei with more than 96 neutrons.

Dramatic departures from simple rotational spacing have been reported for the yrast levels of several even-even deformed rare-earth nuclei. For most of these nuclei, the yrast levels below some critical value of angular momentum can be fitted as members of a ground-state rotational band (grb) with a moment of inertia that increases slowly with rotational frequency. Above this critical value, however, striking changes sometimes occur in the moments of inertia  $\boldsymbol{\vartheta}$  and rotational frequencies  $\boldsymbol{\omega}$  computed from the level spacings. These changes are often referred to as backbending because of the shape of the curve of

 $\theta$  versus  $\omega^2$ .

Pairs of particles moving in a rotating deformed potential experience a Coriolis force which opposes the pairing. Two currently popular explanations of backbending are based on the effect of the Coriolis force on neutron pairs. Calculations based on the early work of Mottelson and Valatin² predict a coherent breakdown of pairing between neutrons at high rotational frequency—a Coriolis antipairing phase transition from a system with the superfluid moment of inertia of the ground state to a system with the larger moment of inertia of a more rigid nucleus. Stephens and Si-

# Prediction of Weak-Coupling Structure from a Shell-Model Basis\*

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Mixed-configuration shell-model wave functions predict the sort of two-nucleon transfer phenomena which has been interpreted as evidence for weak coupling to excited  $0^+$  states.

We present in this note calculations with shell-model wave functions for (0d, 1s)-shell nuclei which indicate that an apparent weak-coupling phenomenon recently observed experimentally in the 0f, 1p shell is, indeed, attributable to weak coupling and, moreover, may be a general feature of the structure of light- and medium-mass nuclei.

The experimental observations we refer to have demonstrated that in several instances<sup>1,2</sup> the (p,t) reaction on odd-mass fp-shell nuclei populates both the ground state and an excited state of the residual nucleus with pure L=0 angular distributions. This establishes, of course, that both the residual states have  $J^{\pi}$  equal to the  $J^{\pi}$  of the target state  $(\frac{7}{2})$  in the examples studied), but the significance of the observations lies in the absence of any general selection rule which would limit the angular momentum transfer in such cases to  $only\ L=0$ . The explanation of this phenomenon thus must arise from specific nuclear-structure properties of the states involved.

The purity of the L=0 transitions to the ground states presumably originates from the dominance of J=0 pairing in forming nuclear ground states. The last, "odd," nucleons of the odd-mass nuclei A + 3 and A + 1 are pictured as weakly coupled spectators to the 0 to ground-state wave functions of the adjacent even-mass nuclei A+2 and A, and the transition between the A+3 and A+1  $J^{\pi}=\frac{7}{2}$ ground states is viewed, essentially, as the pickup of the same correlated J=0 pair by which the A+2 and A ground states are connected. This picture seems empirically verified by the approximate equality of the cross sections of the A+3-A+1 and A+2-A (b, t) ground-state transitions. Weak coupling in such a context is more or less implicitly assumed in all approaches to nuclear structure, just as it is in the very familiar analogous situation involving the "singleparticle" states of a nucleus A+1 and the  $0^+$ ground state of nucleus A.

The question of the extent to which a weakcoupling picture of nuclear structure can be gen-

# Highly Proton-Rich $T_z = -2$ Nuclides: ${}^8$ C and ${}^{20}$ Mg

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Two nuclides with a proton excess of 4 have been produced via the ( $\alpha$ ,  $^8$ He) reaction, and their masses measured. The mass excess of  $^8_6$ C is  $35.30 \pm 0.20$  MeV, and  $^8$ C is therefore unbound with a measured width of  $220^{+}_{-100}^{+10}$  keV. The  $^8$ C mass excess is 0.57 MeV less than the Kelson-Garvey prediction. For  $^{20}_{12}$ Mg, a mass excess of 17.74  $\pm 0.21$  MeV is found, indicating that  $^{20}$ Mg is nucleon stable. Knowledge of the  $^{20}$ Mg mass permits for the first time a test of the isobaric multiplet mass equation in an isobaric quintet, and good agreement is found.

The remarkable accuracy with which the isobaric multiplet mass equation (IMME) describes the masses of analog states is well established. Fifteen complete isobaric quartets  $(T = \frac{3}{2})$  have now been measured, and in only one case, A = 9, is there a small but significant discrepancy—all other  $T = \frac{3}{2}$  multiplets fit the equation precisely. Indeed, the IMME appears to apply even when the states involved are unbound and broad, as in mass 7, a surprising fact in view of the expected presence of level shifts which cannot be entirely absorbed into a quadratic equation. The agreement in mass 7 has been dismissed as fortuitous,1 while the disagreement in mass 9 remains unexplained.2 There is need for an improved understanding of the IMME both through theoretical work and through more revealing experimental tests.

A new and stringent test of the IMME would be the completion of an isospin quintet (T=2). Not only is it less likely that a quadratic equation will fit five masses fortuitously, but the lighter quintets are expected to include both bound and unbound members. At present no more than three members of any quintet are known because all efforts to observe the  $T_z=-1$  and -2 members have been unsuccessful. The observation of isotopes with  $T_z=-2$  (a proton excess of 4) presents a formidable experimental problem because of the apparently small cross sections for every known reaction which might produce such nuclei.

Cerny et al.<sup>3</sup> have observed the reaction <sup>26</sup>Mg( $\alpha$ , <sup>8</sup>He)<sup>22</sup>Mg at  $E_{\alpha}$  = 80 MeV with a cross section of 50 nb/sr, and have used it to obtain the mass excess of <sup>8</sup>He, 31.65 ± 0.12 MeV. However, in a subsequent search for the  $T_z$  = -2 nuclide <sup>20</sup>Mg by

the reaction  $^{24}{\rm Mg}(\alpha,\,^8{\rm He})^{20}{\rm Mg}$ , it was possible only to set an upper limit of 25 nb/sr on the cross section. The apparent evidence for production of  $^{20}{\rm Mg}$  by the reaction  $^{20}{\rm Ne}(\alpha,\,4n)$  observed by Macfarlane and Siivola has since been traced to a spurious instrumental effect.

This Letter reports the observation of two isotopes with  $T_z = -2$ ,  $^8$ C and  $^{20}$ Mg, and the measurement of their masses. The experiments were performed using the Jülich isochronous cyclotron.  $\alpha$  particles of 156 MeV induced the ( $\alpha$ ,  $^8$ He) reaction on targets of natural C and 99.9%-enriched <sup>24</sup>Mg, and outgoing <sup>8</sup>He particles were selected in a double-focusing magnetic analyzer of low dispersion consisting of a dipole element followed by a quadrupole doublet (Fig. 1). The energy of particles accepted by the analyzer was measured with a silicon-detector  $E-\Delta E$  counter telescope. The time taken for particles to traverse the 4.6-m flight path from target to detector was obtained using the cyclotron rf as a reference. By placing four constraints on the particles observed, namely magnetic rigidity, total energy, energy loss ( $\Delta E$ ), and time of flight, background was reduced to an undetectable level. For each event four parameters, E,  $\Delta E$ ,  $E + \Delta E$ ,

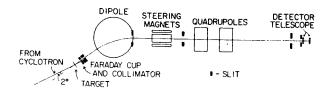


FIG. 1. Schematic diagram of DQQ magnetic analyzer and counter telescope system.

# (p, t) REACTIONS ON ODD-A NUCLEI AND THE WEAK-COUPLING CORE-EXCITATION MODEL

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#### Received 22 February 1974

Excited  $7/2^-$  states to which enhanced L=0 (p, t) transitions are observed have been identified in  $^{41}$ Ca at 2958 keV and in  $^{57}$ Co at 2611 keV. A comparison of these (p, t) transitions with those to the  $0^+$  states in  $^{40}$ Ca at 3353 keV and in  $^{58}$ Ni at 2940 keV leads to the conclusion that these states are obtained by weak-coupling of a  $f_{7/2}$  particle and hole, respectively, to the excited  $0^+$  states of the doubly even cores. It is suggested that these cases are examples of a rather general weak-coupling phenomenon.

Weak-coupling ideas are often invoked in nuclear structure to relate states of neighboring nuclei [1]. Ground states of odd-A nuclei are generally considered as an odd nucleon weakly coupled to the 0+ ground states (g.s.) of the even-even cores. The correctness of this picture is borne out by the large single-particle transfer (spectroscopic factor,  $S \approx 1$ ) found in most such situations. The same picture is often invoked to relate collective 2+ and 3- states in heavy eveneven nuclei with corresponding multiplets, obtained by weak coupling of a single particle, in adjoining odd-A nuclei. The essential requirement for the emergence of such a picture is that the coherence of the core state must be strong enough to be not destroyed by the coupling of the odd particle. The 0<sup>+</sup> ground states  $(0_1^+)$  of even-even nuclei posses this property. The highly correlated nature of the lowest excited 0+ states (02) which occur at modest excitation in most even-even nuclei, generally goes unrecognized unless they can be interpreted as the band heads of a rota-

tional sequence. Recently attention was drawn of these excited 0+ states and their weak-coupling partners in adjoining odd-A nuclei by (p, t) experiments [2]. In a study of the  ${}^{44}$ Ca(p, t) ${}^{42}$ Ca and  $^{45}$ Sc(p, t) $^{43}$ Sc reactions, an excited  $7/2^-$  state was identified at 1.41 MeV in  $^{43}$ Sc by the L = 0 shape of the angular distribution. It was found that both the absolute cross sections and the angular distribution shapes for the transitions to the  $7/2^-$  ground and 1.41 MeV states in <sup>43</sup>Sc were almost identical to those for the transitions to the 0+ ground and 1.84 MeV states, repectively, in <sup>42</sup>Ca. The relative purity of the observed L = 0 angular distributions for  $^{43}$ Sc states (although L = 2, 4, 6 are also allowed), the equality of the cross sections and the smallness of the difference in excitation energies between corresponding states in <sup>42</sup>Ca and <sup>43</sup>Sc were used to suggest that the 1.41 MeV 7/2 state in 43Sc could indeed be regarded as consisting mainly of the 1.84 MeV 0+ state of the core nucleus  $^{42}$ Ca with a  $f_{7/2}$  proton weakly coupled to it, just as the g.s. of  $^{43}$ Sc can be considered as the g.s. of  $^{42}$ Ca with a  $f_{7/2}$  proton weakly coupled to it. The above conclusion, though based on rather dramatic evidence, raises an important question: "Are the observed correlations between the  $(7/2^{-})_{1,2}$  states in <sup>43</sup>Sc and the  $(0^{+})_{1,2}$ 

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# THE EXCITATION OF GIANT RESONANCES IN ELECTRON AND PROTON SCATTERING \*

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Microscopic model calculations are presented for various multipole states in <sup>40</sup>Ca. A resonance-like structure largely due to dipole and quadrupole states is predicted in the giant dipole and quadrupole states is predicted in the giant dipole region with qualitatively the same features observed in electron and proton scattering. The calculated cross-section are in good agreement with those obtained from simple phenomenological collective models, and the available scattering experimental data.

Inelastic scattering of electrons [1,2] and hadrons [3,4] on a wide range of nuclei shows strongly excited resonance-like structures in the low excitation region of the nuclear continuum. The giant dipole resonance (GDR), long known from photonuclear work, has proved inadequate by itself to explain the experimental results, and a giant isoscalar quadrupole resonance (GQR) [3,5] or monopole resonance [5] lying 2-3 MeV below the GDR has been proposed. In addition, other multipoles such as an octopole resonance have been suggested as an explanation for part of the observed strength [1,3]. The resonance-like structures have been widely analyzed in terms of phenomenological collective models, which provide estimates of energy centroid and total available strength (through energy weighted sum rules) for the various multipoles but little information about possible fragmentation of this strength. It is of interest to see what is predicted by a completely microscopic model. A general approach to this problem has been given by Bertsch [6]. In the present paper, however, we consider only a particular case – that of a detailed RPA calculation for <sup>40</sup>Ca [7].

Such calculations give an account of the fragmentation of the multipole strength among states formed by particle-hole excitation, but do not include the further fragmentation among multi-particle multi-hole states. To take this into account, we have simply at-

tributed a spreading width, as in ref. [6], to each RPA state.

We have considered states with  $J^{\pi}(T) = 0^{+}(0)$ ,  $1^{-}(1)$ ,  $2^{+}(0)$ ,  $3^{-}(0,1)$  and  $5^{-}(0,1)$  in the excitation energy region 10-25 MeV. While almost all of the E0, E1, and E5 strength is concentrated in one or two states, the calculations predict many  $2^{+}$  and  $3^{-}$  states. Still, a considerable amount of E2 and E3 transition strength falls in the 10-25 MeV region.

The predictions of the RPA model are as follows:

- (1) Almost all of the isoscalar monopole strength lies in a single state at 14.0 MeV.
- (2) The isovector non-spinflip dipole strength is concentrated in a state at 18.8 MeV, in good agreement with the location of the photonuclear GDR and the main structure of the low momentum transfer (e, e') spectrum. There is some isovector dipole strength in states at 16.5 and 21.3 MeV, the latter state being largely spinflip in character.
- (3) The isoscalar quadrupole strength is contained in a group of states lying between 17 and 21 MeV, dominated by a state at 17.3 MeV.
- (4) The isovector monopole and quadrupole strength is concentrated in the 28-34 MeV region. These states were not considered here.
- (5) The 3<sup>-</sup> strength is fragmented, although some isoscalar 3<sup>-</sup> strength is located in the region of the monopole state.

Table 1 summarizes the B(EJ) values predicted for

<sup>\*</sup> Supported in part by the National Science Foundation.

# TWO-NEUTRON PICKUP STRENGTHS ON THE EVEN LEAD ISOTOPES THE TRANSITION FROM SINGLE-PARTICLE TO "COLLECTIVE"\*

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The (p, t) reaction on  $^{208}Pb$ ,  $^{206}Pb$ , and  $^{204}Pb$  has been studied with 35 MeV protons. It is observed that the L=0 ground-state cross section rapidly increases as one changes targets from  $^{208}Pb$  to  $^{206}Pb$  to  $^{204}Pb$  while the cross section for other natural-parity transitions decreases. The observed cross sections are compared with predictions of the shell model and the pairing-vibration model. Both models are seen to describe some features of the data.

Both the shell model [1, 2] and the pairing-vibration model [3, 4] have been used to describe properties of the lead isotopes. In this note, the predictions of both these models are compared with the observed (p, t) strengths on <sup>208</sup>Pb, <sup>206</sup>Pb, and <sup>204</sup>Pb. These reactions have been studied with a 35 MeV proton beam from the MSU Cyclotron. The tritons were detected by either nuclear emulsions (with resolution of 15 keV, FWHM) or a position sensitive proportional counter [5] (resolution of 30 keV) in the focal plane of a spectrograph. In order to minimize uncertainties in the relative cross sections from isotope to isotope, the identical experimental set up was used to study the (p, t) reaction on all the Pb isotopes. To check these relative cross sections we also studied the (p, t) reaction on a natural lead target for which the ratio of the different isotopes is known. The uncertainty in relative cross section from isotope to isotope is estimated to be less than 8%. Reynolds et al. [6] have studied these reactions with resolution of 220 keV; where similar quantities are reported, our results are substantially in agreement with theirs.

Angular distributions to the lowest 0<sup>+</sup>, 2<sup>+</sup>, and 4<sup>+</sup> states excited in each nucleus are shown in fig. 1

along with the DWBA predictions for these angular distributions. The DWBA calculations used the code DWUCK [7] (in the zero-range approximation) with shell model wave functions describing the initial and final states. Proton parameters from ref. [8] and triton parameters  $(r_0 = 1.16 \text{ fm})$  from ref. [9] were used.

Our main interest in this note is in how the magnitude of the (p, t) cross section changes as one goes away from the 208Pb closed core. In fig. 2 experimentally determined cross sections are compared with the predictions of the shell model for the transitions to 206 Pb and 204 Pb and with the simplest pairing-vibration model (described below) for transitions to the lowest 0+, 2+, and 4+ states. The shell model calculations [1] used a complete six-orbit basis with twobody matrix elements based on those of Kuo and Herling [2]. The theoretical cross sections have been normalized to fit the lowest states of a given L-value observed in the <sup>208</sup>Pb(p, t)<sup>206</sup>Pb reaction; then the same normalization was used to describe the other transitions to states of the same spin and parity. The relative cross section for different L-values is not predicted too well by the shell model; if  $\sigma^{\exp}(\theta)$  =  $N^L \sigma_{\rm DW}(\theta)/(2L+1)$ , then  $N^{L=0}$  is about 70% larger than  $N^{L\neq 0}$ . This suggests that there are correlations absent in the calculated ground-state wave functions which are significant in the two-nucleon transfer pro-

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### COULOMB ENERGY SHIFTS OF THE GROUND STATES OF A = 18 AND A = 42 NUCLEI\*

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### Received 21 March 1974

This work evaluates the theory of Coulomb displacement energies of the ground states of  $^{18}$ Ne- $^{18}$ O and  $^{42}$ Ti- $^{42}$ Ca, with a view to making bounds on the possible violation of charge symmetry in the strong interaction. The theoretical uncertainties are found to be of  $^{8}$ M, and the discrepancies between theory and experiment are much larger than the possible charge-violating potential being sought.

The calculation of Coulomb displacement energies has become the subject of many recent studies carried out in order to resolve the discrepancies between theory and experiment [1-4]. The calculated energy differences of analog states are consistently smaller by ~7% than the experimental values. This problem led to a vigorous study of many, previously neglected, contributions [3, 4], the interplay between the Coulomb forces and the residual interactions, and the charge dependence of the nuclear forces. In the present work we consider the energy differences of the ground states of <sup>18</sup>Ne-<sup>18</sup>O and <sup>42</sup>Ti-<sup>42</sup>Ca applying the best theory available in order to place bounds on the possible violation of charge symmetry of nuclear forces. An appropriate definition for the energy differences, in the case of two valence particles, is given

$$\Delta E = B(T_z = 1) - B(T_z = -1) - 2[B(T_z = \frac{1}{2}) - B(T_z = -\frac{1}{2})]$$
 (1)

where B stands for binding energy. Using the experimental data [5] we find that  $\Delta E = 0.583 \pm 0.01$  MeV for  $^{18}$ Ne- $^{18}$ O and  $\Delta E = 0.425 \pm 0.02$  MeV for  $^{42}$ Ti- $^{42}$ Ca.

There have been several calculations of these two particle Coulomb energy differences in the past [6-9]. However, each calculation neglects some of the correction terms, and there is no assessment of the theoretical reliability of the results. In addition, most of the calculations were carried out using single particle wave functions of inappropriate size and also neglecting the effect of the finite size of the proton charge distribution. \*Supported in part by the National Science Foundation.

The ingredients of the present calculation of the Coulomb interaction between the two valence protons are the following. The framework of the shell model with two particles outside closed shells is adopted using the correct size of the single particle wave functions. The finite size effect of the proton charge distribution, center of mass motion, short range correlations and electromagnetic correction terms are taken into account. The assumed single particle potential well is taken to be the harmonic oscillator using the values of

$$\nu^{-(1/2)} = 1.80 \text{ fm}$$
 for  $A = 18$   
 $\nu^{-(1/2)} = 2.02 \text{ fm}$  for  $A = 42$ 

for the oscillator constant  $\nu = m\omega/\hbar$ . These values in eq. (2) were found [4] to be consistent with the experimental results of the rms radii of the charge distributions of  $^{16}$ O and  $^{40}$ Ca. Also, in obtaining these values of  $\nu^{-(1/2)}$ , the effects of the finite size of the proton charge distribution, center of mass motion and short range correlations on the rms radii of the charge distributions were taken into account.

For the specific wavefunctions we consider two prescriptions for the ground state J=0 T=1 configurations of the two valence particles in A=18 and A=42 nuclei. Pure configurations, of  $|d_{5/2}^2|$  0 for A=18 and  $|f_{7/2}^2|$  0 for A=42, are considered as one extreme, together with the more realistic wavefunctions

$$|\psi(A=18)\rangle = 0.90 \hat{1} |d_{5/2}^2 0\rangle$$
  
+  $0.324 |s_{1/2}^2 0\rangle + 0.287 |d_{3/2}^2 0\rangle$  (3)

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### HOW GOOD IS THE COLLECTIVE MODEL?

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We use Hartree-Fock RPA theories of <sup>208</sup>Pb as a testing ground for collective models of the inelastic scattering transition densities. For isoscalar transitions, the standard collective model and the Tassie model both provide excellent descriptions of the transition density to the strongest states. For the isovector dipole mode, we find two states. The lower state has a transition density resembling the Steinwedel-Jensen model, while the upper state has a transition density resembling the Goldhaber-Teller model. The energetics of the collective states, as summarized in heuristicly derived sum rules, are quite close to the results of the RPA calculations.

We have developed a calculational technique for studying inelastic scattering properties of spherical nuclei, using the RPA response in large particle-hole spaces [1,2] with a Hartree-Fock model of the Skyrme type [3]. Much analysis of data has been done using simple collective models for the response, so it is of interest to compare these models with the more detailed Hartree-Fock RPA theories. The most valuable thing provided by a collective model is the shape of the transition densities between ground and excited states. This transition density is the function  $\rho_{no}(r)$  defined by

$$Y_{M}^{L}(\hat{r}) \, \rho_{no}(r) = \langle \psi_{o} | a^{+}(r) a(r) | \psi_{n}^{L} \rangle$$

where  $a^+(r)a(r)$  is the nucleon density operator at point r, and  $\psi_0$  and  $\psi_n^L$  are ground and excited states of the nucleus.

The simplest collective model for isoscalar excitations, the well-known model of Bohr [4] is based on a classical oscillation of surface position

$$R = R_{o}(1 + \sum_{M}^{L} \beta_{L} / \sqrt{2L+1} Y_{M}^{L}(\hat{r})).$$
 (1)

In this model

$$\rho_{no}(r) = \frac{\beta_L R_o}{\sqrt{2L+1}} \frac{\mathrm{d}\rho_o}{\mathrm{d}r},\tag{2}$$

where  $\rho_0$  is the density of the ground state.

A second model was derived by Tassie [5] under the assumption that the velocity field in the nuclear vibration is irrotational and incompressible (except for L=0). The transition density in the Tassie model is given by

$$\rho_{no}(r) = r^{L-1} d\rho_o/dr, \qquad L \neq 0$$

$$= 3\rho_o + r d\rho_o/dr, \qquad L = 0$$
(3)

This model works very well in fitting shapes of inelastic electron scattering [6], and can also be derived from an energy-weighted sum rule and the assumption that a single state exhausts the sum [7,8].

We calculate particle-hole Green's functions in coordinate space. These are related to transition densities by

$$G(r, r', E) = \sum_{n} \rho_{no}(r) \rho_{no}(r') \left( \frac{1}{E_n - E - i\epsilon} + \frac{1}{E_n + E - i\epsilon} \right)$$
(4)

where the sum is over all excited states of the nucleus. Thus a transition density can be extracted from G by

$$\rho_{no}(r) = \sqrt{\epsilon \operatorname{Im} G(r, r, E_n)}$$
 (5)

or more practically

<sup>\*</sup> Supported by the National Science Foundation.

<sup>\*</sup> These are familiar assumptions in hydrodynamics, so the model is often called the hydrodynamic model. This is a misnomer which becomes evident when the energetics of the vibrations are studied macroscopically [9].

## FAST AQUEOUS CHEMISTRY ON-LINE WITH CYCLOTRON-PRODUCED ACTIVITIES USING A HELIUM-JET RECOIL-TRANSPORT SYSTEM

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A relatively simple system is described that can be used to perform fast on-line chemical separations ( $\leq 1$  s) on activities transported to low background areas in the relatively short times provided by a helium-jet recoil-transport system. The

helium-jet recoil-transport system is run with its low-pressure end at atmospheric pressure, thereby eliminating the need for vacuum pumps and their associated apparatus.

consider aqueous chemistry without having the solutions boiling off. (Actually, we have since discovered

that the HeJRT system can be operated successfully

with its low-pressure end at atmospheric pressure,

chemical separations more or less successfully. Some

have been semi-on-line, others completely on-line and

fast. The most successful to date is the separation of

33-s <sup>63</sup>Ga from 38-min <sup>63</sup>Zn and 24-min <sup>60</sup>Cu, which is

described in some detail in section 2. [A preliminary

version of this separation was presented at the New

York APS Meeting in January 1973 8)]. It now appears

that the numbers and types of different fast chemical

separations that can be performed in conjunction with

a HeJRT system should be essentially unlimited,

making it an even more powerful tool than heretofore

We have been able to carry out a number of different

making chemical procedures even simpler.)

### 1. Introduction

Details of the construction and operation of the Helium-Jet Recoil-Transport (HeJRT) system used in this work are given elsewhere1.2) and numerous references to similar systems also exist<sup>3-7</sup>). Accordingly, we mention only the specific parameters used and the modifications made to the HeJRT system that allowed the performance of fast on-line aqueous chemistry with it. Our HeJRT system presently employs a 14-m long, 0.0014-m i.d. polyethylene capillary between the target assembly and the chemistry apparatus. A small amount of benzene vapor ( $\approx 20$  ppm) is added to the helium supply to generate the large cluster "molecules" which aid in the transport of activities through the system<sup>1,3,7</sup>). The target assembly volume was filled with helium and maintained at a pressure gradient of  $\approx 2$  atm across the system.

The initial hope that "wet" chemistry could be performed on-line with the HeJRT system arose from the observation that the efficiency of the HeJRT system when depositing activities on paper tape did not start to fall off until the pressure in the detector chamber (box where activities transported through the system are normally deposited on some collecting surface and counted) was raised to above 20–30 torr<sup>1</sup>). This was important because the vapor pressure of water at 20°C is 17.5 torr. Accordingly, if it were possible to introduce the activities transported through the system into an aqueous solution, it would be possible to

The first successful chemistry performed with the HeJRT system followed the discovery that not only is it possible to trap activities flowing up the capillary in aqueous solution merely by bubbling the helium flow from the capillary through the solution, but also that the yields remained essentially as good even if the low-pressure end of the capillary and the aqueous solution were at atmospheric pressure. In a series of experiments comparing the amounts of activity trapped in solution under various pressures to that collected on paper tape under  $\approx 1$  torr pressure, it was observed that between 1/2 and 2/3 of the activity could be collected in the aqueous solutions. The amount of activity trapped was largely independent of the pressure over the

realized.

2. Experimental procedures

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<sup>&</sup>lt;sup>†</sup> Work supported in part by the U.S. Atomic Energy Commission.

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## MEASURING NUCLEAR EXCITATION ENERGIES AND *Q*-VALUES WITH A CYCLOTRON-MAGNETIC SPECTROGRAPH SYSTEM\*

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et al.11,12).

Nuclear excitation energies and reaction Q-values have been measured with the M.S.U. cyclotron-magnetic spectrograph system with uncertainties on the order of 1 keV or less. The method involves a spectrograph calibration procedure which utilizes a combination of momentum-matching and kinematics techniques. The calibration lines used in the present work are independent of any previous spectrograph based on the  $^{210}\text{Po}(\alpha)$  energy scale. The present work is also largely independent of

Ge(Li) gamma detector measurements, and provides independent consistency checks on previous measurements at the 1 keV uncertainty level. Sample results include checks on the excitation energies of the first excited state of  $^{12}\text{C}$  and the third excited state of  $^{24}\text{Mg}$ . The excitation energy of the first excited state of  $^{11}\text{C}$  and the Q-values for the reactions  $^{24}\text{Mg}(p,d)^{23}\text{Mg}$  and  $^{24}\text{Mg}(p,t)^{22}\text{Mg}$  are also presented.

The method described in this paper combines momentum matching<sup>9,10</sup>) to determine the beam

energy, kinematics to determine the scattering angle,

and previously known energy levels to determine the

spectrograph calibration. It is possible with the present

method to make independent checks of data based on the  $Po(\alpha)$  standard as well as checks of measurements

made with Ge(Li) gamma detectors. The advantages

and disadvantages of this method of spectrograph

calibration are compared in a later section with the <sup>210</sup>Po alpha calibration used, for example, by Browne

The measurements described in this paper utilized

30-40 MeV proton beams of the Michigan State

University Cyclotron and an Enge split-pole magnetic

spectrograph<sup>13</sup>). Kinematic compensation, as des-

scribed by Enge<sup>13</sup>), is accomplished by moving the

2. Description of experimental set-up

### 1. Introduction

The importance of highly accurate nuclear excitation energies and nuclear reaction Q-values is well established. For example, nuclear structure studies often require the investigation of the levels of a given nucleus via many different experimental techniques. It is the comparison of the properties of a given level in the different reactions that is most relevant in the testing of nuclear theories. Hence the levels of interest must not only be resolved from neighboring levels but also identified unambiguously. Such ambiguity is currently an often recurring problem in the comparison of direct reaction charged particle spectra with the generally much more accurate  $\gamma$ -ray data on the same nucleus. Accurate excitation energies for high lying levels are also useful and sometimes essential in placing unambiguously cross-over  $\gamma$ -rays in a particular decay scheme.

Precise nuclear reaction Q-values or mass differences are also needed for several reasons in nuclear physics. One very active field, for example, is the search for a deviation from the purely quadratic prediction of the isobaric multiplet mass relationship<sup>1,2</sup>). It is now obvious that this requires masses of ground states and of certain excited states with uncertainties of 1 to 5 keV or less<sup>3,4</sup>). Other uses of accurate Q-values are in studies of nuclear Coulomb energy systematics<sup>5,6</sup>), in extracting precise  $\beta$ -decay matrix elements<sup>7</sup>), and in establishing the mass relationship of Garvey et al.<sup>8</sup>) on the proton rich side of the line of nuclear stability.

plate holder to a position which compensates for the kinematic variation of energy with angle  $(dE/d\theta)$ . In addition, by focussing the beam spot such that there is the proper amount of linear variation of energy with position on target some compensation for a finite energy spread in the beam can be made. Using this dispersion matching and kinematic compensation it has been possible to obtain direct reaction spectra with line widths of 3-7 keV fwhm with a beam whose energy spread is many times greater 14,15). In general, how-

ever, conditions are optimum for only one reaction in any given run. Hence in the context of the present work where several different calibration reactions

from several different mass targets are observed

simultaneously certain compromises are necessary.

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## AUTORADIOGRAPHIC LOCALIZATION OF <sup>13</sup>N AFTER FIXATION OF <sup>13</sup>N-LABELED NITROGEN GAS BY A HETEROCYST-FORMING BLUE-GREEN ALGA

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### ABSTRACT

<sup>13</sup>N, generated by proton bombardment of <sup>13</sup>C powder, is rapidly and easily converted to <sup>13</sup>N-N<sub>2</sub>, 0.01 atm pressure, ca. 10 mCi/ml, by automated Dumas combustion. <sup>13</sup>N fixed (as <sup>13</sup>N-N<sub>2</sub>) by algal filaments was localized by an autoradiographic technique which permits track autoradiography with isotopes having short half-lives. Our findings show directly that a minimum of about 25% of the N<sub>2</sub> fixation by intact, aerobically grown filaments of *Anabaena cylindrica* is carried out by the heterocysts. If all of the N<sub>2</sub> fixation takes place in the heterocysts, then the movement of nitrogen along the filaments can be characterized by a constant  $\tau < \text{ca.} 5 \text{ s (cell}^{-2}$ ).

### INTRODUCTION

The site of fixation of molecular nitrogen by aerobically-grown heterocyst-forming blue-green algae has been a matter of controversy. The results of a variety of indirect experiments (Fay et al., 1968; Gorkom and Donze, 1971; Pringsheim, 1968; Stewart et al., 1969; Stewart and Lex, 1970; Weare and Benemann, 1973) have been interpreted as indicating that all of the nitrogen fixation takes place in the heterocysts. However, other observations have suggested that heterocysts are not important for nitrogen fixation: Anabaena flos-aquae was observed to fix nitrogen actively despite a paucity of heterocysts (Kurz and LaRue, 1971); suspensions of Anabaena cylindrica with vegetative cells detached from heterocysts developed nitrogenase activity in less time than was required—in a separate experiment-for new heterocysts to form (Ohmori and Hattori, 1971); and at least

one genus in which heterocysts have not been found fixes nitrogen under aerobic conditions (Rippka et al., 1971; Wyatt and Silvey, 1969), although most such genera do not (Kenyon et al., 1972; Stanier et al., 1971). Heterocysts have been isolated from filaments which had assimilated <sup>15</sup>N-labeled N<sub>2</sub> for 5 min. The concentration of <sup>15</sup>N in the heterocysts was no higher than in intact filaments (Ohmori and Hattori, 1971), but <sup>15</sup>N might have been solubilized from the heterocysts during their isolation. Localization without disruption would be preferable.

Wolk and Wojciuch (1971 a) have shown that heterocysts from sonically disrupted filaments of A. cylindrica have the capacity to account for 25% of the acetylene-reducing (and therefore, presumably, nitrogen-fixing: see Dalton and Mortenson, 1972) activity of intact filaments. We have let algal

### A SHELL-MODEL CALCULATION FOR MASSES 15, 16, AND 17\*

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Shell-model calculations made in the full basis space spanned by the  $0p_{\frac{1}{2}}$ ,  $0d_{\frac{5}{2}}$  and  $1s_{\frac{1}{2}}$  orbits are presented for nuclei with A=15, 16 and 17. The single-particle energies and the two-body matrix elements which constitute the effective Hamiltonian employed in this space were determined by a least-squares fit of the shell-model eigenvalues to 153 experimental energy levels in the A=13-22 region. Results for excitation energies, electromagnetic transition rates, and spectroscopic factors for single-nucleon transfer are presented and compared with existing data. It is found that the observed properties of these nuclei can be understood in qualitative and often quantitative detail in terms of the present calculation.

### I. INTRODUCTION

We report here shell-model calculations for A=15. 16, and 17 nuclei. These results are part of a comprehensive shell-model calculation for all nuclei in the region A=13-24. Results for the heavier nuclei will be published separately [1]. The main purpose of this study is to describe phenomena related to the excitation of one or more particles from the  $0p_{\frac{1}{2}}$  orbit to the 1s,0d shell. Such excitations can give rise to negative-parity states through one or three particle excitations and positive-parity states through two or four particle excitations. The excited states of  $^{16}0$  are the most intensively studied phenomena which arise from configurations of this general type. Brown and Green [2] showed in their pioneering work how much of the salient structure of  $^{16}0$  could be explained with particle-hole combinations which produce deformed states. Other theoretical studies of p-shell to sd-shell excitations have been made by Benson and Flowers, [3] Ellis, Engeland, and Lie [4]. Irvine [5], and Halbert and French [6], Zuker, Buck, and McGrory (ZBM) first showed that a straightforward shell-model calculation, based on a  $^{12}$ C core, could account for much of the  $^{16}0$  structure data in quite accurate detail [7]. The spirit and

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### Summary

We have made an initial computer study of a recycling method for increasing the final energy of heavy-ion beams from ORIC. The system involves simultaneous acceleration of two beams. A first beam originating either in an ion source or external injector is accelerated, extracted, and reinjected in a higher charge state obtained by means of a foil located between the turns of the first-pass beam. example, with a 20 MV tandem injecting into ORIC the energy for <sup>208</sup>Pb after the first pass would be 1.9 or 2.3 MeV/u and after the second pass 5.2 or 6.3 MeV/u depending on the injection charge. The requirements of this scheme are 1) sufficient dee voltage to override the differing mass increase factors of the 2 beams; 2) good beam quality and accelerator stability to insure spatial separation of orbits; 3) an allmagnetic extraction system to assure identical paths for the 2 beams; and 4) relatively standard external beam separation and reinjection equipment.

### Introduction

If a suitable number of electrons are stripped from heavy-ions after acceleration to full energy, the ions can be reaccelerated in the same cyclotron on a different harmonic,  $h=\omega_{\rm rf}/\omega_{\rm orbit}$ .

The energy gain in the second acceleration is given by the square of the harmonic ratio,

$$E_2/E_1 = (h_1/h_2)^2$$
.

In a scheme for double acceleration proposed by Dzhelepov<sup>1</sup> et al., the two beams emerging from the cyclotron were separated by an electrostatic deflector and the primary (first pass) beam was reinjected by stripping inside the cyclotron. This method of injection is currently used at Orsay<sup>2</sup> and at Dubna. Bennett<sup>4</sup> has proposed a double acceleration scheme in which two strippings take place without extraction of the primary beam.

The studies reported in this paper assume the use of a narrow aperture all-magnetic septum<sup>5</sup> to extract both primary and recycled beams along the same path, with velocity selection performed outside the cyclotron. Some of the calculations presented will relate particularly to the Oak Ridge Isochronous Cyclotron (ORIC) as an example.

### Conditions Required for A Recycling System

Several conditions must be met for a successful recycling cyclotron.

(1) The harmonics of the two accelerations and the ion charge states for the first and second accelerations  $(q_1,q_2)$  must meet the condition  $h_1q_1=h_2q_2$ .

- (2) For maximum beam intensity the equilibrium charge state after stripping must be within a few charges of the required charge,  $q_2$ .
- (3) The energy gain per revolution must be sufficiently large so that the phase excursions during the primary and recycle accelerations do not become excessive. This imposes a practical limit on the final energy.
- (4) After stripping the beam must enter the first orbit of the second acceleration period at the proper phase.
- (5) There must be adequate turn separation at the intermediate radius to provide turn separation at the intermediate radius to provide space for the hardware needed for the injection. In this study we have assumed a stripping foil as the injection device.
- (6) Both the initial and recycled beams should arrive at the septum with the same Bp, radius, and preferably radial momentum  $(p_r)$ .

### Practical Configuration

To illustrate the concept of a recycling accelerator a possible configuration is shown in Fig. 1, using ORIC as an example. The source of primary beam can be either an internal ion source or an external source such as a tandem Van de Graaff. If an external source is used the injected beam passes through a stripping foil located inside the cyclotron to achieve the required charge and orbit for acceleration. The primary beam is electrostatically separated from the secondary beam after passing through the magnetic extraction system. A 180° magnet is used to turn the primary beam for reinjection into the cyclotron. A stripping foil, located between turns of the primary beam, achieves the proper charge for reacceleration. The phase of the beam at the start of the second acceleration can be adjusted by changing the path length traveled outside the cyclotron, either by changing the position of the 180° magnet, or by dividing it into three sectors and changing the magnetic fields in the end sections and center section independently. The location of the stripping foil and other parameters can be adjusted so that the first pass and second pass beams both arrive at the entrance with the same Bo, r and  $p_r$ .

### General Characteristics

The energy to which a given mass can be accelerated with a recycling machine has been calculated for several magnet sizes (Fig. 2). The curves shown represent cyclotrons with  $\rm E_{\rm O}$ 's from 50 to 300 where  $\rm E_{\rm O}$  is a constant that characterizes the cyclotron magnet and is defined by the relationship

$$E/u = E_0 \frac{q^2}{A^2}$$

where E/u is the full energy of the machine, in MeV/u, q is the charge state, and A is the mass number.  $E_{o}$ 

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SYSTEMATICS OF "BACKBENDING" PHENOMENA IN THE YRAST BANDS OF 182-188OS
AND OTHER DEFORMED NUCLEI\*

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The discovery by the Stockholm group of the "backbending" effect at high spin has prompted experiments resulting in similar observations for numerous other eveneven deformed nuclei. At least two rather different theoretical explanations have been proposed in an effort to account for the observed effect. To date, however, most experimental data have been acquired for nuclei where the Mottelson-Valetin [Phys. Rev. Lett. 5(1960)511] and the Stephens-Simon [Nucl. Phys.  $\underline{A183}(1972)257$ ] explanations, among others, seem equally applicable.

We have performed a series of experiments utilizing  $(\alpha, xn\gamma)$  reactions in the hope of providing some basis for distinction between the various theoretical pictures of the backbending phenomenon. Self-supporting metal foils of enriched 182,184,195W and  $^{180}$ Hf prepared at the Niels Bohr Institute have been bombarded with 26 and 48-MeV  $\alpha$ -particle beams from the MSU Cyclotron to study the 0s and W isotopes of N=106-112. Gamma-ray singles, angular distributions, excitation functions, and Y-Y coincidence experiments have identified the yrast bands to spin 14-

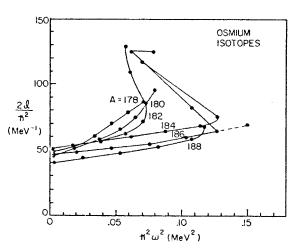


Figure 1: Yrast Bands in Even-Even Os Isotopes

18 in the  $(\alpha,4n)$  products and at least to spin 12 in the  $(\alpha,2n)$  products. The results of the measurements on the osmium isotopic series are shown in Figure 1. Also included for completeness are the 1780s and 1800s curves computed from published data [Nucl. Phys. A92(1967)306].

Three significant conclusions may be drawn from the data shown: 1) A systematic trend in the g.s.b. behavior as a function of spin can be seen from <sup>178</sup>Os to <sup>136</sup>Os.

There is some suggestion that the trend continues in <sup>185</sup>Os.
2) A pronounced increase in the apparent moment of inertia occurs near the critical spin 14 for the yrast states in the <sup>182</sup>, <sup>184</sup>, <sup>186</sup>Os cases.
The most striking effect is seen in <sup>183</sup>Os. 3) Two Y-rays,

both apparently stretched E2 transitions, feed into the spin 12 member of the 1850s g.s.b., a forking in the yrast band hitherto unobserved in nuclei thought to be good rotors. The higher-lying (yrare) branch suggests, both in the excitation function data and in the figure shown here, a smooth continuation of the g.s.b. structure, while the yrast branch is distinct from the g.s.b. in the excitation data and displays the sharpest backbending yet seen in the "rare-earth" region.

Similar data on the yrast band structure have been obtained for the isotopes 130,182W and 154Cd. Data from the latter nuclide can be interpreted to display back-bending behavior in both the g.s.b. and the 3-vibrational band. Both the 185Cg and the 154Cd data suggest that near spin 16, the low-lying bands are intersected by a well-defined higher-lying rotational band. The apparent rotational constant N2/2J for these thus far uncharacterized high-lying bands is 8.0 keV in 197Cg.

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Low-Lying States of  $^{48}V$  from the  $^{48}Ti(p,n\gamma)$  Reaction\*

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The  $^{48}\text{Ti}(p,n\gamma)^{48}\text{V}$  reaction was used with a variety of experiments to study the states of  $^{48}\text{V}$  below 2 MeV of excitation. The WMU Tandem was used for measurements of excitation functions and the MSU Cyclotron for measurements of  $\gamma-\gamma$  coincidences,  $\gamma$ -ray angular distributions, and half-lives in the nanosecond range. In all cases the target was a 0.66 mg/cm $^2$  isotopically enriched  $^{48}$ Ti foil. The experimental techniques and data analyses of these measurements are described elsewhere.  $^1$  The measured half-lives for the 308.3- and 518.8-keV states are 7.12±0.04 and 2.72±0.06 nsec, respectively. No other  $^{48}$ V states below 1.7 MeV of excitation were observed to have half-lives greater than 1 nsec.

Absolute cross sections and the  $\gamma\text{-ray}$  mixing-ratio (\delta) dependent parameters  $A_2^{\star}=A_2/A_0$  and  $A_4^{\star}=A_4/A_0$  were calculated using the statistical compound nuclear (CN) computer code MANDY. Comparisons of experimental relative cross sections (w.r.t. the 308.3-keV 2<sup>+</sup> first-excited state) for the population of each 48V state as a function of proton bombarding energy and of the experimental γ-ray angular distribution  $A_2^{\sigma}$  and  $A_4^{\sigma}$  coefficients with the predictions of the statistical CN theory have then led to the <sup>48</sup>V spin assignments and  $\gamma$ -ray multipole mixing ratios listed in Table I. All transitions indicated were confirmed in the coincidence measurements. The longevity of the 518.8-keV state suggests that the 98.0- and 210.4-keV

 $\gamma$ 's are retarded El transitions yielding a negative parity for this state.

A recent shell-model calculation by Wildenthal<sup>3</sup> using interaction 3 of Vervier<sup>4</sup> and only active  $f_{7/2}$  proton and neutron orbits, has yielded reasonable correspondences with those states suggested to have positive parity. However, extra states appear experimentally at low excitations and are believed to be negative parity states with the  $(\pi d_3/2)^{-1}(\pi f_{7/2})^4(\nu f_{7/2})^5$  or  $(\pi s_{1/2})^{-1}(\pi f_{7/2})^4(\nu f_{7/2})^5$ 

Table I: Energies and Spins of "6V States and Energies, Stanching Ratios, and Multipole Mixing Ratios of "5V y-rays from the "6TI(p,ny) Reaction

configuration. These negative parity states are connected to each other by strong dipole and weak quadrupole y transitions.

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Initial State (keV)	J <sub>i</sub> * <sub>i</sub>	· Jf*f	y-0ut (ke <b>Y)</b>	γ-Branching (Z)		g Ratio
308.3-0.1	2 +	4 +	308.3±0.1	100	E2	
420.8±0.1	1 +	2 +	112.5 ±0.1	100	-0.14	-0.0
427.9:0.1	5(+)	4 +	427.9±0.1	100	-0.15	-0.1
518.8:0.1	1(-)	1 +	98.0:0.1	38 ±2	-0.08	+0.3
		2 +	210.4±0.1	62±2	+0.01	+0.0
613.4:0.1	4(+)	5(+)	185.5:0.1	11 12	-0.02	+0.0
		4 +	613.4:0.1	89±2	-0.21	-0.1
627.3±0.2	6(+)	5(+)	199.3±0.2	37±5	-0.23	+0.0
		4 +	627.3:0.2		63±5 (E2)	
745.1±0.1	2(-)	1(-)	226.3:0.1	92.1	-0.08	-, -0.0
		1 +	324.2:0.1	3.1:0.3	-0.21	+0.1
		2 +	436.8±0.1	4.9:0.4	-0.21	+0.1
765.0±0.1	3(+)	4(+)	151.7:0.2	2.2:0.3	-0.22	
	- • •	2 +	456.7±0.1	55:2	-0.03	-0.0
		4 +	764.9=0.1	45±2	-0.05	+0.0
1055.9±0.2	3(-)	2(-)	310.8:0.1	92±3	-0.03	
		1(-)	537.2:0.2	8±3	(E2)	
1099.3:0.2	4	4 +	1099.3:0.2	100		
1264.6:0.2	5(+)	6(+)	637.3:0.2	25±4	-0.21	+0.2
		4(+)	651.2:0.2	75:4		
1521,5±0.2	2(+)	3(+)	756.4±0.1	20:2	-0.26	-0.0
	- • •	1+	1101.0±0.2		-0.02	+0.1
		2 +	1212.9:0.2	75±4 45±4	-0.05	+0.0
1557.6:0.2	4(-)	3(-)	501.8:0.1	4524 78±4	+0.14	+0.2
		2(-)	812.2:0.3		-0.11	-0.0
1685.6±0.3	(5)	4	586.3:0.2	22±4	(E2)	
1781.0:0.2	3(+)	4(+)	1167.8:0.2	100		<del></del>
	- ( )	2 +	1472.5±0.2	42.5	-0.20	+0.0
		4 +	1760.9:0.3	43 <u>+5</u>	-0.12	+0.0
1998.6:0.2	(3)~	4		15±3		
	1-7-	2(-)	899.4:0.2			
		41~)	1253.3:0.2			-

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### 5.59

THE TENSOR PART OF THE EFFECTIVE INTERACTION FROM N14(p,p')\*
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The  $^{14}\text{N}(p,p')^{14}\text{N}(2.31~\text{MeV})$ ,  $(1^{\dagger}\text{T=0})$ — $(0^{\dagger}\text{T=1})$  transition provides perhaps the best available circumstance to investigate the tensor part of the effective interaction, since the contribution of the normally dominant central interaction is fortuitously reduced by a wave function accident. We have used an Enge split-pole spectrograph to measure cross sections for this transition at bombarding energies of 29.8, 36.6 and 40.0 MeV; the results are shown in Fig. 1. To extract values of the tensor interaction, the data were analyzed using the code EWBA70 (R. Schaeffer and J. Raynal, unpublished) which allows one to include the contributions of knock-on exchange and of central, tensor and spin-orbit forces. The central force  $V_{\rm C}(r)$  had a Yukawa radial dependence and a Serber exchange character. This force, by itself, was entirely unable to reproduce the general trend of the data, especially the forward angle peak. A tensor force of the form  $V_{\rm T}(r)=V_{\rm M}r^2e^{-\alpha t}/\alpha r(\tau_1\cdot\overline{\tau_2})S_{12}$  was added, the relative strengths of  $V_{\rm C}(r)$  and  $V_{\rm T}(r)$  were adjusted to best fit the shape of  $\sigma(\theta)$  and the value of  $V_{\rm T}$  was obtained by normalizing to  $\sigma(\theta)$ . A spin-orbit force with a strength comparable to that used by Love(Phys. Lett. 35B(1971) 371) was also included; its effect was to reduce the cross section by about 15% without appreciably changing its shape. The results are shown in Fig. 1 and Table I. The fits obtained are reasonably good at the lower energies, but do not reproduce the peak which appears near 80° at 36.6 and 40.0 MeV. Reasonable variations of the input parameters do not substantially improve the situation; it may be necessary to invoke an additional reaction mechanism whose importance increases at higher energies.

at higher energies. If one adopts the integral  $J_4 = \int_{\Gamma}^4 V_T(r) dr$  as a measure of the strength of the interaction, one can compare our results with others (see Table I). The present values are substantially larger than estimates based on the one-pion-exchange potential(OPEP) or the Hamada-Johnston potential(HJ). They are also larger than experimental results at lower energies, probably because the present analysis in-  $\widehat{\Gamma}_{105}$  24.8 MeV x125 cludes a spin-orbit force and the exchange

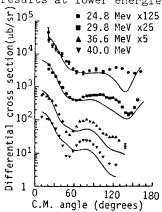


Fig. 1. Cross Sections for  $^{14}\text{N(p,p')}^{14}\text{N(2.31 MeV)}$  The data at  $^{24}$ .8 MeV are from Crawley et al.(Phys. Lett.  $^{328}(1970)92$ ).

E (MeV)	Values of $V_T$ . $V_T (MeV-fm^{-2})^a$	J <sub>4</sub> (MeV-fm <sup>5</sup> )
24.8 29.8 36.6 40.0 OPEP HJ(r <sub>c</sub> =0	19.2 16.3 13.3 15.0 .6fm)	555 470 383 432 318 294

amplitude for the tensor force, both of

which reduce the calculated cross section.

- a. For 1/α=0.816 fm. See S.M. Austin, The Two-Body Force in Nuclei (Plenum Press, 1972 p. 285.
- b. See the reference of note a.
- † Research supported by the National Science Foundation

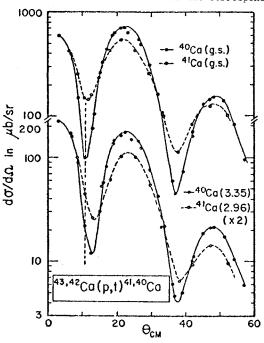
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5.118

(p,t) REACTIONS ON ODD-A NUCLEI AND THE CORE-EXCITATION MODEL
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In a recent paper Seth et al. (Fhys. Rev. Lett. 30 (1973) 132) have reported a 'text-book example' of the weak-coupling core-excitation model in a study of the reactions 45Sc(p,t)43Sc and 44Ca(p,t)42Ca at 52 MeV. They used the selection rules for two-neutron transfer reactions to identify an excited 7/2 state at 1.41 MeV in 43Sc through the characteristic L=0 transfer. It was found that both the absolute cross sections and the angular distribution shapes for this transition are almost identical with those for the L=0 transition to the 1.84 MeV 0½ state in 42Ca (as are the corresponding g.s. transitions). These observations were used to suggest that the 1.41 MeV 7/22 state in 43Sc could indeed be regarded as consisting mainly of the 1.84 MeV 0½ state of the core-nucleus 42Ca with a f7/2 proton weakly coupled to it. We have studied the reactions 43Ca(p,t)41Ca and 42Ca(p,t)40Ca to answer the important question raised by the 45Sc, 44Ca experiments, viz.: "Are the observed relationships between the 7/2 states in 43Sc and the 04 states in 42Ca essentially accidental or are they expressions of a more general phenomenon which may be expected to occur between other even and odd nuclei of the f7/2 shell?" If the latter is true, following the example of 43Sc and 42Ca, we expect the 7/2 core-coupled state (coupled to the 3.35 MeV, 0½ state in 40Ca) to occur in 41Ca at = 3.35 - 0.4 = 2.95 MeV.

The (p,t) reactions on  $^{43}$ Ca and  $^{42}$ Ca targets were studied using the 42 MeV proton beam of the Michigan State University cyclotron and the split-pole spectrograph equipped with a single-wire proportional counter. Energy resolution, FWHM=30 keV was realized. As shown in the adjoining figure an L=0 transition was indeed found to the 2.96 MeV state in  $^{41}$ Ca establishing its  $J^{\Pi}=7/2^{-}$ . None of the other proposed L=3 states in the  $(^{3}$ He, $\alpha)$  experiment of Lynen et al. (Phys. Lett. 25B (1967) 9) were found to have any identifiable L=0 components, making it unlikely that any of them has  $J=7/2^{-}$ . Unlike the  $^{43}$ Sc- $^{42}$ Ca states, there are identifiable differences between the "L=0 transitions" to the corresponding states in  $^{41}$ Ca and  $^{40}$ Ca. The shapes of the transitions for  $^{41}$ Ca



shapes of the transitions for  $^{41}\text{Ca}$  states indicate presence of small L=2 components. Further, the  $^{41}\text{Ca}$  (g.s.) transition is only 65% as strong as the  $^{40}\text{Ca}$  (g.s.) transition and the  $^{41}\text{Ca}$  (2.96 MeV,  $7/2^-$ ) transition is only 26% as strong as the  $^{40}\text{Ca}$  (3.35 MeV,  $0^+_2$ ) transition. On the other hand, the characteristic differences between  $f^2$  and  $d^2$  pick-up seen between  $f^2$  and  $d^2$  pick-up seen between  $f^2$  Ca (g.s.,  $0^+_1$ ) and  $f^2$  (3.35 MeV,  $0^2$ ) transitions are faithfully reproduced in the corresponding  $f^2$  (g.s.,  $f^2$ ) and  $f^2$  (2.96 MeV,  $f^2$ ) transitions.

Other examples of (p,t) experiments on odd-A nuclei where similar weak-coupling (to excited 0<sup>+</sup> state of the core) picture is suggested will also be discussed.

\*The MSU Cyclotron project is supported in part by the National Science Foundation.

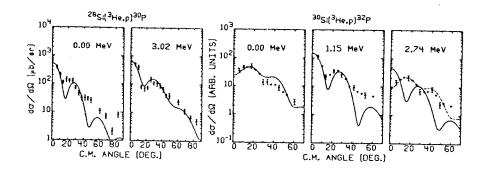
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Study of the (3He,p) Reaction on the Si-Isotopes\*

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As a part of our systematic investigation of the ( $^3\text{He,p}$ ) reaction on nuclei in the 2s-1d shell we report here the results for the  $^{28,29,30}\text{Si}(^3\text{He,p})^{30,31,32p}$ reaction. The experiments were performed with the <sup>3</sup>He beam from the MP tandem accelerator and the multigap magnetic spectrograph of the Max-Planck-Institut für Kernphysik in Heidelberg. The <sup>28,30</sup>Si(<sup>3</sup>He,p) reactions were measured at 28 MeV bombarding energy, the <sup>29</sup>Si(<sup>3</sup>He,p) reaction at 26 MeV. We have tried to reproduce the experimental differential cross sections with DWBA calculations which employ spectroscopic amplitudes extracted from shell model wave functions. For the analysis of the <sup>28,29</sup>Si(<sup>3</sup>He,p)<sup>30,31</sup>P reaction only one set of wave functions was available whereas for the <sup>30</sup>Si(<sup>3</sup>He,p)<sup>32</sup>P reaction two different sets of wave functions were used. In the adjoining figure we show as an example measured and calculated angular distributions of transitions to 1+ states in <sup>30</sup>P and <sup>32</sup>P. In the  $^{28}\text{Si}(^{3}\text{He},\text{p})^{30}\text{P}$  reaction these angular distributions show mostly L=0 patterns with some L=2 admixture. The shapes are quite well predicted by the shell-model wave functions although the prediction of the magnitude of the differential cross section for the 3.02 MeV transition is much too small. Contrary to this, one observes for most of the transitions to 1<sup>+</sup> states in the <sup>30</sup>Si(<sup>3</sup>He,p)<sup>32</sup>P reaction a predominantly L=2 character. This behavior is again qualitatively reproduced by the wave functions. For the transitions to the 0.00 and 1.15 MeV states, both sets give the same shape of the angular distributions, but slightly different magnitudes. For the 2.74 MeV transition one set predicts a predominantly L=0  $\,$ pattern whereas the other one a nearly pure L=2 structure in agreement with the experimental data. In the  $^{29}Si(^{3}He,p)$  reaction the agreement between the experimental data and the theoretical cross sections for transitions to states with excitation energies lower than 4.3 MeV is quite good. The only exception is the transition to the second  $1/2^+$  state in  $^{31}P$  at 3.14 MeV. The experimental data show a mixture of L=0 and L=2 contributions of nearly equal strength, which is not reproduced by the shell-model wave functions.

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### 5.329

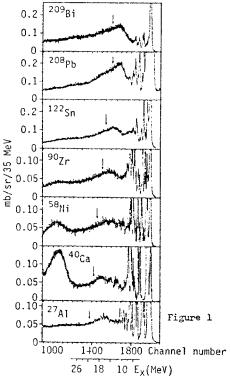
Observation of the Giant Quadrupole State by High Energy Inelastic <sup>3</sup>He Scattering\*

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A giant resonance at about 2 MeV of excitation below the giant dipole resonance (GDR) has been observed by both inelastic electron scattering (Nagao and Torizuka, Phys. Rev. Letters 30, 1071(1973)) and by inelastic proton scattering (Lewis and Bertrand, Nucl. Phys. A196, 337(1972)). However recent <sup>3</sup>He inelastic scattering results are conflicting. (Lewis, Phys. Rev. C7, 2041(1973) and Peterson, Nucl. Phys. A202, 557(1973)). In the present experiment, the inelastic scattering of 71 MeV, <sup>3</sup>He particles from a number of nuclei from <sup>27</sup>Al to <sup>209</sup>Bi was studied using both a solid state detector telescope and a wire proportional counter in the focal plane of an Enge Spectrograph. Both methods gave consistent results. Spectra at 20° from the detector telescope are shown in Fig. 1. A strong yield is observed to excitation energies 2-3 MeV below the GDR, the position of which is shown by an arrow. The peak at channel 1000 is H(<sup>3</sup>He, <sup>3</sup>He).

After subtracting a linear background and the GDR, angular distributions were obtained for the remaining peak (33 MeV wide). Figure 2 shows angular distributions from <sup>197</sup>Au and <sup>208</sup>Pb, together with theoretical predictions for L=2 and L=0 transfers (Satchler, to be published). While the shape of the angular distribution cannot be used to distinguish the L transfer, the cross section is almost 10 times the energy weighted sum rule (EWSR) limit for L=0, but is consistent with the EWSR limit for L=2. Thus the high energy <sup>3</sup>He scattering does appear to confirm the existence of a giant quadrupole state over a broad region of mass number.

\* Work supported by the National Science Foundation.



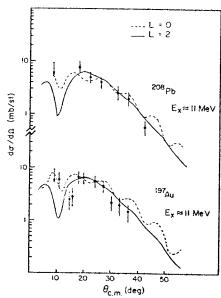


Figure 2

9.2

Exploring Z>N Nuclei with Multinucleon Transfer Reactions\*

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Of the approximately 55 known nuclei with Z>N, about 13 have been discovered or studied by means of the (3He, 6He) reaction. The experiments which used this reaction at Michigan State University were all performed in a split-pole spectrograph. The detection techniques which have evolved over the years included a single position resistive silicon detector, a telescope of such detectors, a photographic plate with specially designed absorbers, and a wire-counter plasticscintillator combination. The figure below illustrates the method of detecting and measuring the energy of the <sup>6</sup>He's as it is presently employed. A key ingredient is time-of-flight particle identification, which has proven to be the best method of separating out this low cross section (typically 1  $\mu b/sr$ ) reaction from the very strong competing processes. An example of the type of results obtained is the first observation of  $^{55}\rm{Ni}$  and also a mass-measurement with 15 keV accuracy and a determination of the level structure. The nuclei which have been studied over the past few years include

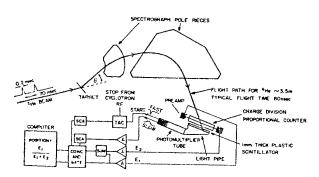
- Mass and Energy levels of  $^{21}\mathrm{Mg}$ ,  $^{25}\mathrm{Si}$ ,  $^{37}\mathrm{Ca}$ ,  $^{43}\mathrm{Ti}$ ,  $^{47}\mathrm{Cr}$ ,  $^{51}\mathrm{Fe}$ , and  $^{55}\mathrm{Ni}$ .1,2,3
- II. Mass of Ground State: 90, 130, 295.1,4
- III. Search for new Energy Levels: 6Be, 10C, 24Al.5

Future work using the same techniques will center on the (p,6He) reaction which has been shown to have cross sections within the feasible range for the method. $^{f 6}$ 

 $\star$  Work supported by the National Science Foundation.

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TIME OF FLIGHT PARTICLE IDENTIFICATION IN THE SPECTROGRAPH

Paper 15

BACK-BENDING PHENOMENA IN 154Gd AND 182-1880s

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The doubly even nuclei 154cd and 182-1880s have been studied by means of the (a,2n) and (a,4m) reactions, using 24-50 MeV a beams from the MSU Cyclotron. By means of Y-ray singles, Y-Y coincidence, excitation function and angular distribution measurements with Ge(Li) detectors, we have identified the yrast members to spin 14-18.

In  $^{154}$ Cd, both the ground and  $\beta$ -bands were identified to spin 18. A plot of the moment of inertia, I, vs. the rotational frequency,  $\omega^2$ , yields back-bending curves in both these bands. The data may be interpreted in terms of a third band which intersects the  $\beta$ - and ground-bands in the vicinity of spins 12 and 16, respectively.

In the yrast states of 182,184,1860s, a pronounced back-bending is observed near spin 14. Two spin 14<sup>+</sup> members exist in <sup>186</sup>0s, both feeding into the 12<sup>+</sup> level. The higher-lying member appears to be a smooth continuation of the ground bend, whereas the lower one is part of the 'back-bent' yrast branch which appears to be an extension of another band that has now intersected the ground band.

While the observed back bending seems consistent with a band-crossing picture, it is not clear whether the intersecting band is the decoupled two quasiparticle band of Stephens and Simon [Nucl. Phys. A183, 257 (1972)] or a band in which the pairing correlations of the nucleus have been destroyed.

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### The $^{154}$ Sm( $\alpha$ , $^{6}$ He) $^{152}$ Sm Reaction $^{\dagger}$

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The  $(\alpha, ^6\text{He})$  reaction is intermediate between light and heavyion processes, and it may be a gateway to the understanding of more complex heavy-ion reactions. It may also be a useful complement to the (p,t) reaction for nuclear spectroscopy. Two previous studies [1,2] have been performed on light nuclei, therefore we chose to study a heavier nucleus via the  $^{154}\text{Sm}(\alpha,^6\text{He})^{152}\text{Sm}$  reaction. In this deformed region, multistep processes are often important in the reaction mechanism, and we can gain insight into the role of such processes in heavy ion reactions.

The experiment was performed using 49.7 MeV  $\alpha$ -particles from the MSU cyclotron. The emergent  $^6{\rm He}$  particles were analyzed in an Enge spectrograph. Details of the  $^6{\rm He}$  detection system are found elsewhere [3].

Data were taken at eight angles,  $\theta_{L=4^{\circ}}$  to 20°. Fig. 1 shows the angular distributions obtained for the 0<sup>+</sup>, 2<sup>+</sup> and 4<sup>+</sup> members of the ground band. Some qualitative features determined are: the ground state,  $\ell=0$  angular distribution resembles a (p,t) distribution, although its features are compressed; in general, the cross sections are smaller than in (p,t); high  $\ell$ -transfers are not favored; the

favored  $\ell$ -transfer is  $\ell$ =2. These last features are due to the momentum match of the  $(\alpha, {}^6\text{He})$  reaction in this case.

The small cross sections of the higher excited states do not allow us to determine if the shapes of these angular distributions are characteristic of a given 1-transfer value.

We have made no attempt as yet to fit the data using DWBA or CCBA with realistic form factors. We have done simple DWBA calculations using a single-configuration form factor. We have had moderate success in fitting the \$l=0\$ angular distribution with this. In addition there is fair agreement between the calculation and the data for the first 2<sup>+</sup> state, which is a better result than one obtains for (p,t) at the same stage of approximation.

Supported by the U.S.A.E.C. and the N.S.F.

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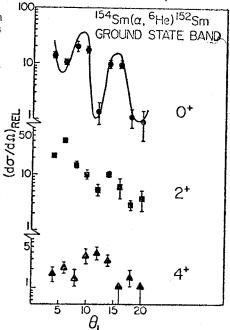


Fig. 1.--Angular distributions for the observed members of the ground band. The cross sections are relative, and the errors shown are statistical. The line through the 0<sup>†</sup> distribution is merely to guide the eye.

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HELIUM JET RECOIL TRANSPORT METHOD USED FOR STUDYING NUCLIDES FAR FROM BETA STABLILITY  $^\dagger$ 

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The Helium Jet Recoil Transport (HeJRT) method has distinct advantages when used to look at short lived activities far from beta stability. This is a result of its continuous nature and its low transport time ( $\approx 0.2$  sec). The HeJRT method also has an advantage over pulsed beam techniques in that the activities are counted in a low background area remote from the target, and a moving collector system easily removes unwanted daughter activities.

Progress has been made in increasing the versatility of the HeJRT system. Towards this end a new target assembly with drop-in components has been constructed. The components are easily interchanged to allow for different modes of operation of the HeJRT system<sup>2</sup>: normal-continuous flow, compound-continuous flow, pulsed flow, etc. Similarly, targets are easily inspected and changed during a run. An added feature of the target assembly is a multiple absorber assembly providing remotely controlled absorber thicknesses from 0.004 to 0.200 inch of Al in 0.003 inch steps. A multiple target assembly is under construction to increase yields.

Target preparation has also been simplified. Metal powders are cemented onto aluminum backings, and oxides are pressed onto back-

ings with a pressure of 2250 tons/sq.in.

Fast aqueous chemistry has been conducted enroute to the detector to enrich the activity of interest. 3 The flow of helium from the HeJRT system containing the activities is mixed with a flow of an aqueous solution and forced through an ion exchange column, performing separations in <1 sec. In addition, partial separations have been achieved in the 0.2 sec transit time by use of different trace impurities in the carrier gas to preferentially transport those activities of interest.

An 8-channel digital sequencer has been added to the system to control the movement of the tape onto which the activities are sprayed by the jet, and to perform other tasks such as pulsing the accelerator or routing spectra within the computer for half-life determinations.

The system has been tested by identifying 143Gd as having halflives of 119 sec and 41 sec for the metastable and ground states, respectively. The HeJRT techniques are currently being applied to an extensive  $\gamma-\gamma$  coincidence study of this activity.

We are now in the final stage of construction of an extention of our system patterned after R.D. Macfarlane's MAGGIE system4 that will allow the on-line mass identification of activities by a timeof-flight measurement in coincidence with the  $\gamma$ -spectrum.

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