Recoil-order Corrections in β-decay

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The least ambiguous tests for second class weak interactions involve measurements of angular correlations, such as βγ or βα, in nuclear beta decay.[1] With inclusion of recoil to first order in q/m, (momentum transfer divided by nucleon mass), the βγ correlation depends on a linear combination of the weak magnetism (b) and tensor (d) form factors. To higher order, other form factors contribute as well.[1] Thus, to determine the second-class tensor form factor, all other contributing terms must be included in the analysis of precise allowed β-decay measurements.

We are using one-body transition density matrix elements obtained from large-basis shell-model calculations to evaluate the main nucleon (b,d and higher order) form factors as given by the impulse approximation.[1] From these calculations it should be possible to interpret individual βγ correlation of e^+ and e^- decays when they become available. While the validity of the impulse approximation can be questioned for the higher-order recoil terms, the extended shell model had been shown to work quite well for calculating the leading term i.e., the Gamow-Teller matrix element.[2] It may be interesting to see if this is also the case for the higher order terms.

The main purpose of the calculation, however, is to make a reasonable estimate of the size of the higher-order form factors to determine if they have an important influence on the determination of the second-class tensor term. We also evaluate the form factors which affect the shape of the beta spectrum with the goal of assessing the possibility of determining the weak magnetism form factor from measured β-spectrum shape. The combination of measurements of the spectral shapes, the βγ correlation and the calculation of higher-order terms should provide useful information on the validity of the CVC theory and the possible existence of second class currents.

The analog e^+, e^- decays of the mass-20 system have been investigated; results show that higher order effects are not negligible. In Fig. 1, we illustrate the contributions from various terms to the βγ correlation for mass-20 system; the weak magnetism term (b) only; $b+d^I$ (first class tensor); and finally with higher order terms included. Note that the quadratic energy dependence (due to higher order terms) is significant, particularly for ^{20}Na. The spectrum shape factor for ^{20}F also shows a noticeable quadratic effect but the main contribution comes from the weak magnetism term. Other cases of particular interest, such as the mass-19, 22, 24, 32, and 35 systems in the sd-shell region, are also being investigated.

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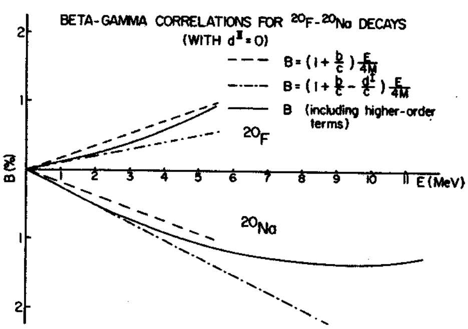

The Empirical $(1f_{7/2})^n$ Model

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A dominant subset of the states in the nuclei with $20 \leq N, Z \leq 28$, which span the region from ^{40}Ca to ^{48}Ca to ^{56}Ni, stand out in the context of the nuclear shell model as being remarkebly well described by protons and neutrons filling a single j-orbit, namely $1f_{7/2}$. The first extensive $(1f_{7/2})^n$ shell-model calculations were carried out by McCullen, Bayman and Zamick.[1] Since their paper experimental information about $1f_{7/2}$ shell nuclei has increased tremendously, especially for the high-spin yrast structures. Thus we have undertaken a complete calculation of $1f_{7/2}$ shell observables with $(1f_{7/2})^n$ wave functions. Complete tabulations of excitation energies for all states as well as E2 and M1 matrix elements and wave functions for the first and second excited states have been made. Calculation of the β-decay matrix elements and spectroscopic factors (one-#andtwo-nucleon transfer) are planned.

The effective interaction for the eight $(1f_{7/2})^2$ two-body matrix elements are obtained directly from the "two-particle" nuclei ^{42}Ti, ^{42}Sc, ^{42}Ca, ^{46}Ca, ^{48}Sc, ^{50}Ti, ^{54}Fe and ^{54}Co. In particular, the calculation has been carried out with the following five sets of two body matrix elements which we believe are most relevant (See Table I):

42SC-INT) The particle-particle interaction extracted from the lowest level of each spin in ^{42}Sc.

54CO-INT) The hole-hole interaction extracted from single levels (not necessarily the lowest of each spin) in ^{54}Co.

48SC-INT) The Pandya transformed particle-hole interaction from the lowest level of each spin ^{48}Sc.

42SC*-INT) The particle-particle interaction from the centroids of levels in ^{42}Sc weighted with the spectroscopic factors from the $^{41}Ca(^{3}He, d)^{42}Sc$ reaction.[2]

48SC*-INT) The proton-nuetron interaction from ^{48}Sc and the proton-proton and neutron-neutron interaction from single levels in ^{46}Ca.

Here we give two examples of the results and comparison with experiment; ^{48}V energy levels and B(E2) values for the $6^+ \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0^+$ cascades. The excitation energies of levels in ^{48}V are given in Table II calculated with three different interactions and from the experimental data summarized in Ref. 3 and obtained recently by Samuelson.[4] The comparison of experiment and the calculation with the 42SC-INT is very good especially for the highest spin states. The calculation with the 48SC-INT gives a much too compressed spec... the high-spin states even though ^{48}Sc and ^{48}V are closer to each other on the nuclear chart than ^{42}Sc and ^{48}V. This situation is improved by using the 48SC*-INT in which $\langle V_{pn} \rangle \neq \langle V_{nn} \rangle$ and $\langle V_{pp} \rangle$ for J=0, 2, 4 and 6. A similar conclusion concerning $\langle V_{pn} \rangle$ and $\langle V_{pp} \rangle$ was obtained by Serduke[5] for the $(1g_{9/2})^n$ configurations.

The best overall values for the effective single particle transition operators needed to fit the E2 and M1 observables are effective charges of $e_p = 1.9$ and $e_n = 0.9$ and effective g-factors of $g_p = 1.456$ and $g_n = -0.377$. The B(E2) values for the $6^+ \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0^+$ yrast cascades are given in Table III. The B(E2) values were calculated with the 42SC-INT; the wave functions and hence the B(E2) values are relatively insensitive to the choice of interaction. The comparison with experiment is especially good for the high-spin states where the admixture of other configurations can be accounted for with a state independent effective operator. Many of the $2^+ \rightarrow 0^+$ transitions including those in the middle of the $1f_{7/2}$ shell are enhanced relative the calculation, whereas the N=28 $2^+ \rightarrow 0^+$ transitions are in good agreement with the theory.

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Table I Input data for the $<1f^2_{7/2} \mid V \mid 1f^2_{7/2}>_{JT}$ matrix elements (MeV)

J^π (T)	42SC-INT [^{42}Sc]	54CO-INT [^{54}Co]	48SC-INT [(^{48}Sc)$_T$]	42SC*-INT [^{41}Ca(^{3}He,d)]	48SC*-INT [(^{48}Sc)$_T$+^{46}Ca]	
0^+ (1)	-3.174	-2.528	-2.068	-2.89	-2.068	-2.675
1^+ (0)	-2.563	-1.591	-2.061	-1.68	-2.061	
2^+ (1)	-1.588	-1.082	-0.757	-1.10	-0.757	-1.329
3^+ (0)	-1.683	-0.250	-0.989	-0.82	-0.989	
4^+ (1)	-0.357	+0.102	+0.119	-0.26	+0.119	-0.100
5^+ (0)	-1.663	-0.377	-0.811	-0.80	-0.811	
6^+ (1)	+0.063	+0.557	+0.333	+0.07	+0.333	+0.299
7^+ (0)	-2.556	-2.329	-2.218	-2.56	-2.218	

Table II Excitation energies of levels in ^{48}V

J (π=+)	Exp. Refs. 3,4 (keV)	$(\nu 1f_{7/2})^5$ $(\pi 1f_{7/2})^3$ Theory 42SC-INT	48SC-INT	48SC*-INT
4	0	0	0	0
2	308	-155	67	42
1	421	297	301	283
5	428	607	428	410
4_2	613	277	228	228
6	627	465	228	334
3	765	772	880	826
7	1255	1177	761	963
2_2	1522	1519	1574	1687
8^2	2231	2319	1631	2008
9	2626	2670	1759	2200
11	(4306)	4421	2977	3732
10	(4396)	4439	3240	3962
12		6817	5068	6129
13		6729	4665	5722
14		8620	6354	7842
15		9340	6600	8088

Table III B(E2) values for the even-even 6+→ 4+ →2+→ 0+ yrast cascades

Nucleus	$J_i \to J_f$	$B(E2)_{exp}$ (e^2fm^4)	$B(E2)^a_{th}$ $(1f_{7/2})^n$	$B(E2)^{b,c}_{th}$ $(fp)^n$
^{42}Ca	2→ 0	83.4 ±2.8	15.4	8.0
	4→ 2	65 ±7	15.4	8.0
	6→ 4	6.42± 0.11	7.0	4.0
^{44}Ca	2→ 0	96± 7	21	12.9
	4→ 2		23	14.2
	6→ 4	41.9± 2.8	20	11.9
^{46}Ca	2→ 0		16.4	11.7
	4→ 2		16.3	11.7
	6→ 4	5.34 ±0.28	7.4	4.8
^{42}Ti	2→ 0	150 ±25	69	30
	4→ 2	<230	68	30
	6→ 4	28 ±3	31	15.1
^{44}Ti	2→ 0	117 ±25	107	50
	4→ 2	280 ±60	135	70
	6→ 4	160 ±20	59	46
^{46}Ti	2→ 0	186 ±8	116	d
	4→ 2	180 ±20	128	d
	6→ 4	150^{+160}_{-80}	110	d
^{48}Ti	2→ 0	150± 10	101	91
	4→ 2	100 ±20	126	114
	6→ 4	54± 5	76	69
^{50}Ti	2→ 0	53 ±8	77	66
	4→ 2	60 ±12	77	66
	6→ 4	33.8 ±1.2	35	32
^{48}Cr	2→ 0	260^{+150}_{-70}	152	153
	4→ 2	330 ±180	184	186
	6→ 4		187	189
^{50}Cr	2→ 0	208 ±23	136	d
	4→ 2	160 ±20	176	d
	6→ 4	130 ±30	142	d
^{52}Cr	2→ 0	113 ±10	106	102
	4→ 2	83 ±17	117	113
	6→ 4	60 ±3	97	94
^{54}Fe	2→ 0	107 ±8	81	88
	4→ 2	77 ±16	81	88
	6→ 4	40.7± 0.7	37	37

a) $\hbar\omega=41\ A^{-1/3}$, $e_p=1.9$ and $e_n=0.9$

b) The $2p_{3/2}$ and $1f_{5/2}$ orbitals are included via perturbation theory see text, next section.

c) $\hbar\omega=41\ A^{-1/3}$, $e_p=1.10$ and $e_n=0.56$

d) Not yet calculated

Effective Operators in j^n Configurations--
Two-body Contributions to the Transition Operators in the $1f_{7/2}$ Shell

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In a recent paper,[1] the effective E2 matrix elements for the nuclei surrounding ^{16}O and ^{40}Ca were investigated using wave functions which explicitely included all $\Delta N = 0$ and $\Delta N = 1$ configurations (N refers to a major harmonic oscillator shell) in a Woods-Saxon basis. In this work it was found that effective charges of $e_p = 1.06 \pm 0.07e$ and $e_n = 0.57 \pm 0.03e$ were needed for the $1f_{7/2}$-$1f_{7/2}$ matrix elements. The enhancement over the E2 matrix elements of the bare operator, Fig. (a), is due to contributions from the $\Delta N=2$ giant quadrupole states, which may be represented diagrammatically as in Fig. (b). The $\Delta N=2$ effective charge is expected to have a very smooth orbital and mass dependence since the giant quadrupole states should have similar properties in all nuclei. The effective operator for the combination of the diagrams in Figs. (a) and (b) will be represented by ∿∿o as Fig. (c) and the matrix elements of this effective operator are denoted by $Q(J_i, J_f, n) = \langle (1f_{7/2})^n J_f || E2^{eff} || (1f_{7/2})^n J_1 \rangle \times (2J_1+1)^{-1/2}$.

We have investigated the structure of the effective operator which should be used for calculations in which the shell-model basis must be truncated even within a major harmonic oscillator shell, in particular for the $(1f_{7/2})^n$ configurations. For n=1 there are only $\Delta N=2$ contributions. For n=2, in addition, the $\Delta N=0$ contributions shown in Fig. (d) are present which represent in this case the virtual excitation of the $j=1f_{7/2}$ particle to the $j'=2p_{3/2}$, $1f_{5/2}$ or $2p_{1/2}$ orbits. The value of this diagram depends on the initial and final spins and will be denoted by $\delta Q(J_i, J_f, n=2)$. It can easily be shown that if the energy denominator for δQ is approximated by the single-particle energy difference $\Delta\epsilon(j') = \epsilon(j)-\epsilon(j')$, then $\delta Q(J_i,J_f,n)$ for $n>2$ is just the linear combination:

$$\delta Q(J_i,J_f,n>2) = \frac{n(n-1)}{2} \sum_{J_o J_i' J_f'} [j^{n-2}(\alpha_o J_o) j^2 J_f' | \} j^n(\alpha_f J_f)] \times [J^{n-2}(\alpha_o J_o) j^2 J_i' | \} J^n(\alpha_i J_i)] \times (-1)^{J_o+J_i'+J_f+k} \sqrt{(2J_i+1)(2J_f+1)} \begin{Bmatrix} J_f' & J_f & J_o \\ J_i & J_i' & k \end{Bmatrix} \times \delta Q(J_i', J_f', n=2)$$

where Q is a tensor of rank k=2 and [|}] are two-body coefficients of fractional parentage. This relation can easily be generalized for the isospin formalism or for several model-space orbitals as in the case of the p-n formalism.

Thus, up to first order in perturbation theory the $\Delta N=0$ contributions to the j^n transition matrix elements can be represented by an effective two-body tensor operator whose matrix elements are all in principle related to measureable quantities in the two-particle system.

When the j-shell is nearly filled it is convenient to express the effective operator in terms of the hole representation, in which case the effective two-body tensor operator transforms into an effective one-plus two-body operator. The one-body contribution, shown in Fig. (e) for n=2j, has the familiar property that it multiplies the bare operator by a constant, $\delta Q(J_i=J_f=j, n=2j) = Q(J_i=J_f=j, n=2j)\delta e^{\Delta N=0}$, and in general has the value $Q(J_i,J_f,n)\delta e^{\Delta N=0}$. δQ for the m-hole configuration, Fig. (f), can then be simply related to δQ for the m-particle configuration, $\delta Q(J_i,J_f,n=2j+1-m) = (-1)^k \delta Q(J_i,J_f,n=m) + (-1)^{k+1} Q(J_i,J_f,n=m)\, \delta e^{\Delta N=0}$. An interesting special case of this relation is for the middle of the shell where $\delta Q[J_i,J_f,n=(2j+1)/2] = Q[J_i,J_f,n=(2j+1)/2]\delta e^{\Delta N=0}/2$.

E2 matrix elements have calculated for the $1f_{7/2}$ nuclei using the method described above and the $(1f_{7/2})^n$ wave functions described in the last section. The two-body tensor matrix elements $\delta Q(J_i,J_f,n=2)$ were calculated with the renormalized Kuo-Brown interaction[2] and with single-particle energies $\Delta\epsilon(2p_{3/2})=2.1$ MeV and $\Delta\epsilon(1f_{5/2})=6.5$ MeV. Harmonic oscillator wave functions with $\hbar\omega = 41\, A^{-1/3}$ MeV were used and an orbital independent $\Delta N=2$ effective charge was assumed.

A least squares fit to twelve high spin transitions from A=42-54 yielded $e_p \approx 1.10$ and $e_n \approx 0.56$; these are in good agreement with the values obtained in our previous investigation.[1] The calculated B(E2) values for the $6^+ \to 4^+ \to 2^+ \to 0^+$ cascades are given in Table III of the previous section. The overall agreement with experiment is not improved much from the pure $(1f_{7/2})^n$ calculation; however, the effective charges needed are much smaller than the values of $e_p \approx 1.9$ and $e_n \approx 0.9$ required for the pure $(1f_{7/2})^n$ calculation.

The largest contributions from the $\Delta N=0$ excitations come from the T=0 p-n interaction. For example, the most important enhancement for the $(\pi 1f_{7/2})^2$ ^{50}Ti transitions arises from neutron core excitations, see Fig. (g). For the ^{50}Ti $6^+ \to 4^+$ transition $\sqrt{B(E2)} = Q + \delta Q = 3.76e_p + 2.78e_n$ compared with the pure $(1f_{7/2})^2$ value of $3.12e_p$.

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