Transport Theory of Deeply Inelastic Heavy Ion Collisions

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In heavy ion collisions at center-of-mass energy 5-10 MeV/A, the reaction cross section is dominated by the deeply inelastic reaction in which a large fraction of the relative kinetic energy and relative angular momentum are transferred into other degrees of freedom. Also many nucleons are exchanged between the two ions. To describe this type of nuclear reaction, we have developed a transport theory which is based on the multiple distorted wave Born approximation and statistical assumptions on the coupling matrix elements.

We use the following model Hamiltonian

$$H = -\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + U(r) + H_o(\zeta) + V(\vec{r},\zeta)$$

where μ is the reduced mass and $U(r)$ is the sum of Coulomb and real nuclear potential. $H_o(\zeta)$ is the intrinsic Hamiltonian and is assumed to have eigenstates given by $H_o(\zeta)|s,x\rangle=\epsilon_s|s,x\rangle$, where $x=\frac{Z_1-Z_2}{Z_1+Z_2}$ is the charge asymmetry variable. The part $V(\vec{r},\zeta)$ couples the relative motion with the intrinsic degrees of freedom.

In terms of the matrix element $V_{sx,ty}(\vec{r})$ $\langle s,x|V(\vec{r},\zeta)|t,y\rangle$, the triple differential cross section of energy, angle and charge asymmetry can be expressed as a Born series. Due to the high excitation energy of the intrinsic degrees of freedom during the reaction, the intrinsic states involved are complex when expressed in terms of simple shell-model states. Furthermore, experimentally only a group of these states are observed, it is therefore sufficient to consider only the average value of the differential cross section. This is achieved by introducing a Gaussian ensemble of $V_{sx,ty}(\vec{r})$ with zero mean and a second moment given by

$$\overline{\langle s,x|V(\vec{r},\zeta)|s',x'\rangle\langle s',x'|V(\vec{r}',\zeta')|s'',x''\rangle}$$

$$=\delta_{ss''}\delta(x-x'')w_o\sqrt{D_sD_{s'}}\exp\left(-(\vec{r}-\vec{r}')^2/2\sigma^2\right)\times$$

$$\times\exp\left(-(\epsilon_s-\epsilon_{s'})^2/2\Delta^2\right)\times\frac{1}{\sqrt{2\pi\delta}}\exp\left(-(x-x')^2/2\delta^2\right)f\left(\frac{r+r'}{2}\right)$$

Here W_o is the strength of the coupling, D_s is the level spacing, σ measures the spatial correlation between two matrix elements, while Δ and δ are essentially the energy and charge asymmetry transfer per collision, respectively. The function f describes the dependence of the matrix element on the separation of the two ions. Based on the picture of a potential well due to one nucleus impinging into another nucleus, the parameters have been estimated for $^{40}Ar+^{208}Pb$ to be $\Delta\sim 7$MeV, $\sigma\sim 4$fm, $w_o\sim 20$MeV, and f has an exponential dependence on distance similar to that obtained from the overlap of two density distributions.[1]

Theoretical techniques[2] have been developed to evaluate the ensemble averaged cross-section for any strength of the coupling, and it is given by the asymptotic value of the average density matrix $\overline{\rho}_{sx}(\vec{R},\vec{R}')$. The Wigner distribution function $F_{sx}(\vec{R},\vec{k})$ of $\overline{\rho}_{sx}(\vec{R},\vec{R}')$ satisfies a transport equation of the Vlosov type

$$\left\{\frac{\hbar^2}{2\mu}\vec{k}\cdot\vec{\nabla}_R-\vec{\nabla}_R U\cdot\vec{\nabla}_k\right\}F_{sx}(\vec{R},\vec{k})$$

$$=\sum_t\int dx'\int d^3\vec{R}'\int d^3\vec{k}'\{G_{sx,tx'}(\vec{R},\vec{k};\vec{R}',\vec{k}')F_{tx'}(\vec{R}',\vec{k}')$$

$$-L_{sx,tx'}(\vec{R},\vec{k};\vec{R}',\vec{k}')F_{sx}(\vec{R}',\vec{k}')\}$$

where G and L are related to the second moment of $V_{sx,ty}(\vec{r})$.

It has been shown in a one-dimensional model[3] that the solution of the above transport equation can be approximated by taking the classical limit of the distribution function, i.e. $U(r)+\frac{\hbar^2k^2}{2\mu}+\epsilon_s=E$ where E is the total energy, and by considering only the first and second moments of the distribution function.

The model has been applied to study various deeply inelastic heavy ion collisions.[4] The nuclear potential used is the proximity potential of Swiatecki et al., while the Coulomb potential is that between two point charges. The form factor f is calculated fom the overlap of two density distributions. Parameters used are $\delta=\frac{0.4}{(Z_P+Z_T}$ $\Delta=7$MeV, $\sigma=4$fm, and $W_o=17.5$ MeV. In Figs. 1 and 2, we show the angular distribution of the two reactions $^{136}Xe+^{209}Bi$[5] and $^{40}Ar+^{232}Th$[6] at laboratory energy 1130 MeV and 388 MeV respectively. It is seen that the calculated results agree reasonably with experiments. Especially, the characteristic differences between the two experimental cross sections are well reproduced.

In conclusion, our transport theory is capable of describing the deeply inelastic heavy ion collisions. However, in view of more detailed experiments, the model should be further studied both theoretically by checking the various approximations used and phenomenologically by finding a better potential and an improved parameterization of the second moment of the coupling matrix elements.

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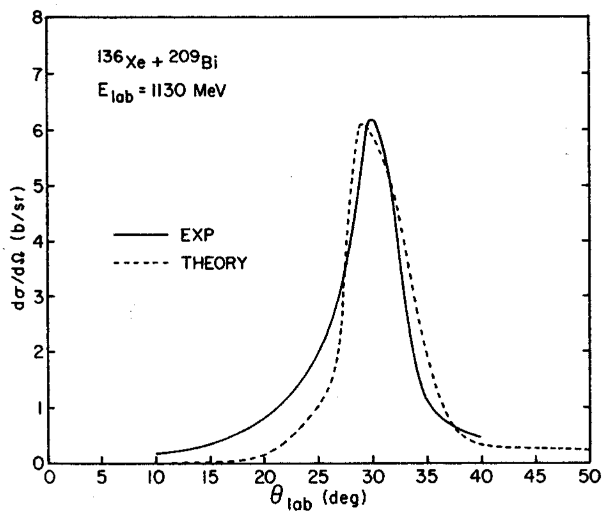

FIG. 1.--The differential cross section in b/sr as a function of laboratory angle for the reaction $^{136}Xe+^{209}Bi$ at laboratory energy 1130 MeV.

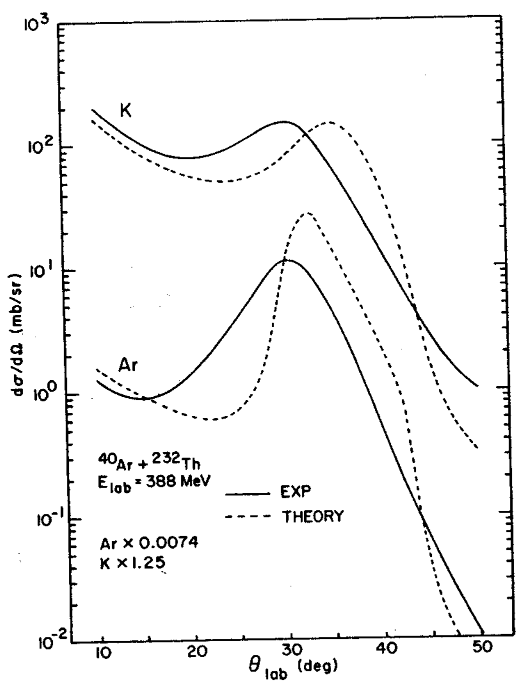

FIG. 2.--The differential cross section in mb/sr of light products as a function of laboratory angle for the reaction $^{40}Ar+^{232}Th$ at laboratory energy 388 MeV.

The time-dependent Schrödinger equation was applied to study the energy of nucleons emerging from heavy ion collisions. The model is based on one-dimensional potential wells. The initial condition is an eigenstate of a potential well, and the collision is represented by the approach of a second potential well. The two wells move together until their centers are separated by a certain distance R_{min}. At this point the wells are frozen. This represents the fusion of the two nuclei.

The particle wavefunction is no longer an eigenstate, and in fact there is an appreciable probability for the particle to be ejected from the combined system. As an example, a test case was investigated with the initial well of the form:

$$V(z) = \frac{-50 \text{ MeV}}{1+e^{(z-2.88)/0.65}} \qquad (1)$$

A neutron wave function with one node is bound by 18.2 MeV in this well. This represents the least bound p orbit in ^{14}N; the total energy would also include the kinetic energy in the other two directions which amounts to

$$\frac{1}{2}\hbar\omega \sim 7.5 \text{ MeV}.$$

The target well was the same form as eq. (1). The probability of the neutron emerging with more than 27.5 MeV in the lab, is shown in Fig. 1 as a function of R_{min}. The bombarding energy is 10 MeV/A above the Coulomb barrier. Notice that the probability is about 1% even for large values of R_{min}. In effect, the particle is shaken off by the stopping of the well. For very small values of R_{min}, the particle hits the farther edge of the target well before it has stopped, and the probability increases.

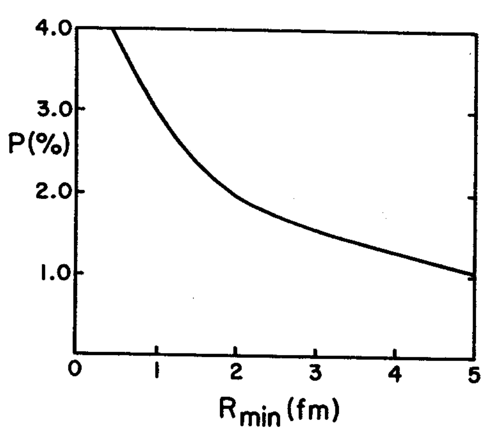

Fig. 1.--Probability of emission of a nucleon with energy greater than 27.5 MeV in the lab, from an orbit bound by 10.6 MeV, as a function of the distance of approach of the two heavy ions. The bombarding energy is 10 MeV/A.

In Fig. 2 is shown the probability of the neutron emerging with energy greater than 27.5 MeV, as a function of lab bombarding energy. It is a rather steep function which approaches 10% at E_{lab} = 20 MeV.

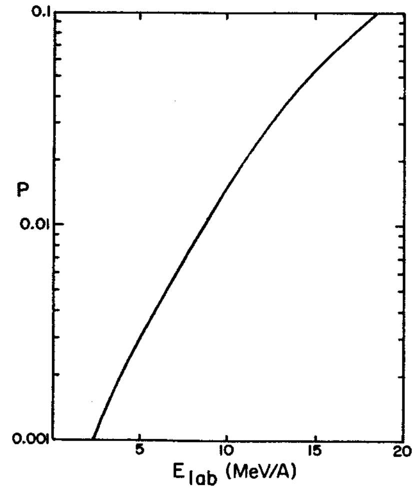

Fig. 2.--Probability of emission of a nucleon with energy greater than 27.5 MeV in the lab, as a function of the bombarding energy of the projectile. The distance of closest approach of the two ions is 3 fm.

Other mechanisms producing energetic particles will make it difficult to measure this effect. One interfering effect from quasielastic collisions is the decay of projectile excitations by nucleon emission. For example, when the bombarding energy is 10 MeV/A, a nucleon emitted with 4 MeV from the projectile can have a lab energy exceeding 27.5 MeV. Thus it would be necessary to exclude the nonfusion reactions from the measurement. Also, the high energy tail of the statistical distribution of evaporation nucleons is significant with collisions on light targets. The distinguishing feature from the statistical model would be the forward peaked angular distributions, and the dependence of the rate on bombarding energy.