A Nuclear Cascade Program

G. Bertsch

I have constructed a program for cascade calculations of heavy ion collisions. The model should have a range of validity covering projectile energies from 150 MeV/n to 2 GeV/n. The lower limit is set by the approximations in the cascade, namely the neglect of the potential field and the Pauli principle in the dynamics. The upper limit is set by the parameterization made for nucleon-nucleon collisions and pion production. I used a parameterization similar to ref. 1, treating pions as Δ excitation of nucleons. To make the calculation fast, the algorithm evolves the system by alternately updating the position and momentum vectors of all the particles. For time steps 0.5 fm/c, the error in the evolution of the classical equations is inconsequential.

The input of the program includes the masses of the colliding nuclei, the radii of the nuclei, the momentum per nucleon of one of the nuclei in the c.m. frame, the time interval per step for the numerical evolution, and the number of time steps. The output is a list of the final coordinates of the nucleons, giving their momentum, position and masses. The mass differs from 0.938 GeV when the nucleon is excited to a Δ state.

The program takes 4 1/2 minutes to calculate a collision of mass 208 on 208, running on our computer which has a speed comparable to the PDP 10. The program consists of 214 FORTRAN statements, and will be provided to interested users upon request.

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1. J. Cugnon, J. Mizutani, and J. Vandermeulen, Nucl. Phys. A352, 505 (1981).

Global Angular Distributions in the Cascade Model

G. Bertsch

The angular distribution of particles emerging from heavy ion collisions provides information on the multinucleon dynamics at high density. In particular, the final state momentum distribution from central collisions is quite sensitive to the degree to which the system attains local equilibrium. Hydrodynamic models, which are based on the assumption of local equilibrium, predict a side-splash of particles from head-on collisions.

We are now calculating the angular distributions the cascade model to compare with the hydrodynamic predictions. We consider collisions of $^{40}Ca+^{40}Ca$ at 800 MeV/n bombarding energy. As a measure of the shape of the final state momentum we define the two parameters,

$$\alpha_0 = \frac{\sqrt{4\pi}}{N}\sum_i^N Y_{20}(\theta_i,\phi_i) = \frac{\sqrt{5}}{N}\sum_i^N \left(\frac{3}{2}\cos^2\theta_i - \frac{1}{2}\right) \qquad (1)$$

$$\alpha_1 = \frac{\sqrt{4\pi}}{N}\left|\sum_i^N Y_{21}(\theta_i,\phi_i)\right| = \left|\sqrt{\frac{15}{2}}\,\frac{1}{N}\sum_i^N \sin\theta_i\cos\theta_i e^{i\phi_i}\right| \qquad (2)$$

In these equations, the sums are over N particles measured in the final state, with angles measured from the mid-velocity frame. The parameters α_0 and α_1 were computed for the hydrodynamic model in ref. 1, and it was found that α_0 varied from -0.5 at zero impact parameter to +2.0 for grazing collisions. The parameter α_1 has a peak at mid impact parameter, $\alpha_1 \sim 0.7$, falling to zero at the extremes.

The details of the cascade model are described elsewhere. We only include particles that have been struck in evaluating the sums in eq. (1-2). We find that the average values of α_0 and α_1 do not display as much variation with impact parameter in the cascade model as they do in hydrodynamics. The comparison of the cascade model with hydrodynamics for α_0 is shown in Fig. 1. The line represents the hydrodynamic prediction,

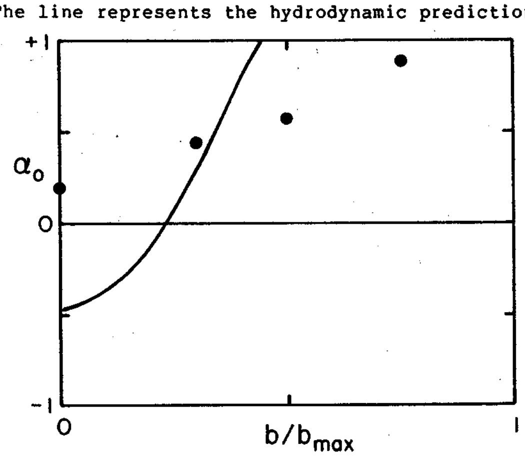

Fig. 1. Predicted angular distribution coefficient α_0 for the reaction $^{40}Ca+^{40}Ca$ at 800 MeV/n. The solid line is the hydrodynamic prediction from ref. 1, and the dots are from the present cascade calculation, averaged over 10 runs.

and the points are cascade results, averaged over ten runs, and including all 80 particles each time in eq. (1). Note that the different definitions of α_0, using all particles for the hydro, make comparison of the two models only sensible for the smaller impact parameters. We see that the cascade prediction always has positive α_0, i.e. a prolate momentum distribution along the beam axis. This result has been obtained in other cascade calculations.[2] The r.m.s. deviation of the individual values of α_0 does not appear to be a good way to determine the impact parameter. In fact, a simple counting of the number of participants has a dispersion at $b/b_{max} = 0.5$ of 10% of its variation, while α_0 at the same impact parameter has a dispersion of 20% of its variation. To distinguish the two models, it would be desirable to restrict the events to impact parameters $b \le \frac{1}{4} b_{max}$. This could be done by including only the 4% of the events having the highest multiplicity. With this cut on the data, a positive α_0 with a value in the range $\alpha_0 \approx 0.3$ would show that independent collision dynamics persist at high density. Conversely, a low or negative value of α_0 would demonstrate the presence of collective effects.

We next turn to the azimuthal asymmetry of the momentum distribution, measured with the parameter α_1. The dependence of α_1 on impact parameter is shown in Fig. 2. For head-on collisions the mean value of α_1 is 0.07, while for $b = 0.5\, b_{max}$ the mean value is 0.27. Again, the dispersion in α_1 is rather large compared to the variation, with the r.m.s. deviation of individual runs being $\alpha_1 = 0.08$. The hydrodynamic prediction for α_1 is much higher. Restricting the events to $b \le \frac{1}{2} b_{max}$ by using a multiplicity cut, the hydrodynamic model predicts $\alpha_1 \approx 0.6$.

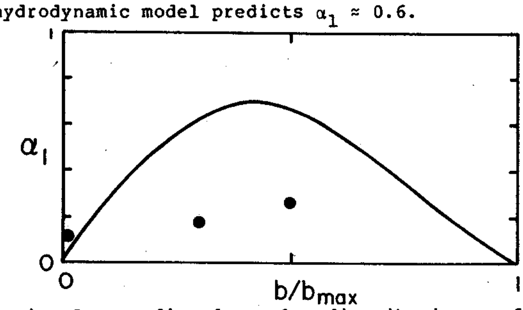

Fig. 2. Predicted angular distribution coefficient α_1, with solid line and dots the hydrodynamic and cascade results, as in Fig. 1.

We conclude that it would be easier to distinguish the models by the predicted azimuthal asymmetry (α_1) than by the predicted side-splash (α_0).

1. G. Bertsch and A.A. Amsden, Phys. Rev. 18, 1293 (1978).
2. J. Cugnon, T. Mizutani and J. Vandermeuten, Nucl. Phys. A352, 505 (1981).

Central Region Clustering in the Cascade Model

G. Bertsch

In a previous study,[1] the final state clustering in head-on heavy ion collisions was calculated in the cascade model. The number of deuteron-like pairs is given in the sudden approximation by the expression,

$$N_d = 12\, N_p \langle f \rangle \qquad (1)$$

Here N_p is the number of protons, including those contained in composite particles, and $\langle f \rangle$ is the average phase space occupation probability in the vicinity of the protons. This is calculated as

$$\langle f \rangle = \frac{\int d\gamma f^2(p,r)}{\int d\gamma f(p,r)} \text{ where } \int d\gamma = \frac{d^3p d^3r}{(2\pi h)^3} \qquad (2)$$

The occupation factor f is normalized as usual to f = 1 for a completely occupied region of phase space. For the case we studied, $^{40}Ca + ^{40}Ca$ collisions at 800 MeV/n and zero impact parameter, we found $\langle f \rangle \approx 0.07$, which leads to considerable clustering in the final state, $N_d \approx N_p$. I.e., roughly half of the particles are predicted to be bound in composites such as d, t, ^3He and alpha particles. This is contrary to experiment, which shows that the fraction of deuteron-like pairs is only 30%, of the particles emerging in the central region.[2]

In this work we report the result of similar calculations for noncentral collisions. Details of the cascade model are described separately. We find that $\langle f \rangle$ is much lower for particles produced from grazing collisions. This is to be expected: the nucleon-nucleon collision process averages f over a region of coordinate space of the order of the mean free path. Particles in the nuclear surface effectively have a lower density in phase space, when they are ejected to a different region of phase space by the collision process.

We first show, in Fig. 1, the fraction of particles participating in collisions in the cascade model, which is called z. This is shown for various runs of the cascade program by points in the figure. This may be compared with the curve showing the geometric overlap of target and projectile, assuming no motion in the transverse direction. The number of participants in the cascade tends to be somewhat larger, due to secondary collisions induced by particles knocked out of the overlap region. The average participant fraction measured in inclusive experiments is

$$\bar{z} = \frac{\int_0^{b_{max}} z\, b_{max} db_{max}}{\frac{1}{2} b_{max}^2} \qquad (3)$$

This fraction is 30% in the cascade model, compared with 24% for the geometric overlap.

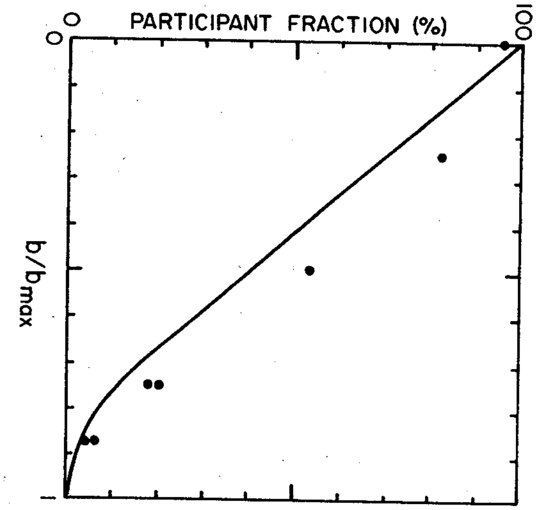

Fig. 1. The fraction of struck particles in $^{40}Ca + ^{40}Ca$ collisions is plotted as a function of impact parameter. The solid line is calculated in the geometric overlap model, and the points are results of the cascade model, averaged over 10 runs.

We next calculate $\langle f \rangle$ as a function of impact parameter. The distribution function f is founded by counting participant particles in cells of phase space, as discussed in ref. 1. The result is shown in Fig. 2. We see that $\langle f \rangle$ decreases by roughly a factor of 3 as the impact parameter

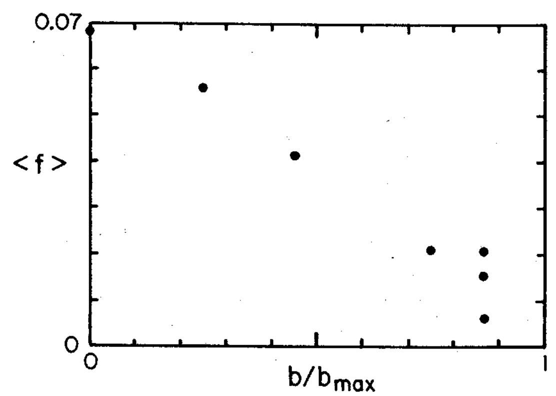

Fig. 2. The mean phase space occupation factor in the final state of collisions of $^{40}Ca + ^{40}Ca$ at 800 MeV/n bombarding energy. The points are calculated in the cascade model, averaging over 10 runs. Note the large fluctuations for near-grazing collisions.

is increased from zero to b_{max}. The experimental data of ref. 3 makes no selection on multiplicity, so we should compare average values of N_d with N_p. These may be calculated from

$$N_p = \bar{z}(Z_t + Z_p) \qquad (4)$$

$$N_d = 12\bar{N}_p\langle f\rangle \qquad (5)$$

with

$$\langle\bar{f}\rangle = \frac{\int_0^{b_{max}} \langle f\rangle_b zbdb}{z} \qquad (6)$$

We find $\langle\bar{f}\rangle \simeq .041$. This gives a deuteron-cluster to proton ratio of 0.5, which is still higher than the experimental ratio. However, the discrepancy is now much smaller, and it is possible that the approximation implicit in eq. (1) could be responsible.

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1. G. Bertsch and J. Cugnon, to be published in Physical Review C.
2. P. Siemens and J. Kapusta, Phys. Rev. Letters 43 (1979) 1486.
3. S. Nagamiya, et al., Berkeley Report LBL 12123 (1980). See Table IV.

Random Walk Approach to Deeply Inelastic Heavy Ion Reactions

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A constantly swelling mass of deeply inelastic heavy ion reaction data has been accumulated in recent years. These data are often analysed in the framework of the transport theory with the use of the Fokker-Planck equation. While the transport theory provides qualitative and sometimes quantitative description of the deeply inelastic process, a complete description of the data requires the introduction of many other degrees of freedom.[1] Another approach was proposed by Griffin et al.[2] and by Busch and his collaborators[3] where the importance of the potential energy surfaces of nuclei is stressed. In their approaches a nucleon may move freely from one nucleus to the other if the two nuclei touch together with the probability governed by their binding energies. This random walk approach has a large success even under a very simplified assumption[2] in explaining the mass distribution of the deeply inelastic process. These two groups, however, fail to reproduce the energy scale, i.e. the system relaxes too quickly for a given amount of energy loss. It seems to us that this deficiency arises from the fact that the particle transfer is only the process to dissipate the kinetic energy and to transfer it to the intrinsic energy. Hence, we propose here to introduce another mechanism which transfers energy but no particles between the two nuclei. This process may be caused by interactions between nucleons in different nuclei; this is usually called 'frictional' damping.

We initially consider the probability for particle transfer and the energy dissipated in the friction process as free parameters and intend to calculate not only the (N,Z) mass distributions but also the angular distributions for each element. The first calculation of the mass distribution was performed by adding the frictional damping process to the procedure of Griffin et al.[2] The calculated results look promising for the charge the mass variances although the drift curve in the (N,Z) plain comes out to be unsatisfactory as has been reported by Griffin et al.[2] We are now proceeding to use realistic surface energies, as used by Busch,[3] and to introduce the frictional damping process, where the use of the realistic energy surface should result in the desired drift curve. The challenge in our approach is then to compare the calculated angular distributions for each element to experiments.

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1. W. Narenberg, Proceedings of Predeal International School on Heavy Ion Physics, Bucharest (1978) p. 825 and references therein.
2. J.J. Griffin, Y. Boneh, K.-K. Dan and M. Dworzecka, preprint ORO 5126-126, University of Maryland, p. 81-135 (1981).
3. F. Busch, preprint (1981) University of Marburg; D. Schull, W.S. Shen, H. Freiseleban, R. Bock, F. Busch, D. Bange-t, W. Pfeffer and F. Puplhofer, preprint (1981); D. Schull, private communication.