NUCLEAR STRUCTURE
(THEORETICAL)
The effect of interactions on level densities is a poorly understood subject. The level density is usually parameterized in terms of the Fermi gas model, which involves as a parameter the single-particle density of states. The empirical values of this parameter,

\[ g = \frac{4}{5} \frac{dn}{dE} \text{ MeV}^{-1} \]

differs from the infinite Fermi gas value by a factor of 2. It has been argued that the difference can be ascribed to static surface effects, based on the Thomas Fermi model.\(^2\) It is easy to show, however, that static surface effects only increase the parameter by about 25%. The argument is based on the formula for the single-particle level density in a potential well derived by Siemens\(^3\) and others. Describing the well by a function \(V(R, r)\), where \(R\) is a measure of the radius of the well (for example, the distance where the potential is half its maximum), the formula is

\[ g(E_F) = \frac{2\pi}{2\pi} \frac{dV}{dr} \left( \frac{2\pi^2}{3} \right)^{1/3} \left( \frac{4\pi R^2}{d(E_F) - 2} \right) \text{ MeV}^{-1} \]

where \(d(k)\) is the phase shift of a wave with momentum \(k\) along the normal to the surface, and the phase shift is measured with respect to a sine wave that vanishes at \(R\).

If the potential radius is defined so that \(d(k_F) = \frac{2\pi}{k_F}\), then there is no surface contribution to the level density and the naive Fermi gas formula can be applied. In Fig. 1 we show \(d(k)\) for the usual Woods-Saxon potential,

\[ V(R, e) = -50 \text{ MeV} \]

\[ 1 + e^{-\frac{R - 0.55}{0.45}} \]

Fig. 1. Phase shift of bound state in semi-infinite Woods-Saxon potential, as a function of energy. The energy is measured with respect to zero binding for states with no momentum in the plane of the surface.

The phase shift goes through \(\frac{\pi}{2}\) near the Fermi energy; thus the usual Woods-Saxon well radius gives the correct volume for the Fermi gas level density. Substituting \(R = 1.25 a^{1/3}\) from Eq. (1) yields

\[ g(E_F) = \frac{4}{11.4} \text{ MeV}^{-1} \]

The momentum dependence of the Brueckner-Hartree nuclear potential reduces this by about 25%, so an effective mass enhancement of about a factor of 2 is needed to explain the empirical parameter. Since calculations of effective mass based on perturbation theory only give enhancements of 1.25-1.5, there remains a serious discrepancy.

We are presently investigating whether self-consistent Hartree-Fock theory provides a better treatment of level densities. We first calculate a ground state in Hartree-Fock theory together with the associated single-particle spectrum. The excited states are constructed by changing the occupation numbers of the single-particle states, and renormalizing the Hamiltonian. We expect that this will give a much compressed spectrum, compared to the independent particle model.

SURFACE RESPONSE IN NUCLEI

H. Esbensen and G.F. Bertsch

The collective response of finite nuclei has been studied in great detail in the random phase approximation (RPA) using a shell-model representation. In order to obtain some of the global features of the surface response, avoiding the detailed dependence on shell structure, we study the response of a semi-infinite slab of nuclear matter. The free response of such a system has previously been used to study the importance of single-step processes in high energy inelastic proton scattering on heavy nuclei.¹

The self-consistent RPA equation for the induced density $\delta \rho$ due to an external field $V_{ext}$ is

$$\delta \rho(\vec{r}, \omega) = -\int d\vec{r}' G_0(\vec{r}, \vec{r}', \omega)[V_{res}(\vec{r}') + \delta V(\vec{r}', \omega)]$$

where $G_0$ is the field-free Green's function and

$$\delta V(\vec{r}, \omega) = \int d\vec{r}' v(\vec{r}, \vec{r}') \delta \rho(\vec{r}', \omega)$$

is the induced potential originating from the residual interaction $v$. The formal solution is expressed in terms of the RPA Green's function:

$$\delta \rho(\vec{r}, \omega) = -\int d\vec{r}' G_{RPA}(\vec{r}, \vec{r}', \omega)V_{res}(\vec{r}')$$

For an infinite system, as for example a free electron gas, this equation is easily solved in Fourier space. For finite nuclei it can be solved numerically on a finite grid in coordinate space.²

For a semi-infinite system with a diffuse surface, which we consider, it can in general be difficult to obtain a solution. The problem is that a perturbation in the surface region will propagate into the interior of the slab, and a large residual interaction can lead to a strong coupling of bulk density oscillations and the surface response.

For a separable residual interaction, however, the RPA response of a semi-infinite slab can be reduced to closed integrals that can be evaluated numerically. In the figure we show the isoscalar and isovector response, together with the free response. Here we have chosen the separable residual interaction

$$v(\vec{r}, \vec{r}') = \frac{2\pi}{2} \int d\vec{r}_2 G_{RPA}(\vec{r}, \vec{r}_2, \omega) G_{RPA}(\vec{r}_2, \vec{r}', \omega)\rho(\vec{r}_2)$$

where $\rho_0$ is the single particle potential for the slab, and the strength $\kappa$ has been chosen to simulate a realistic isoscalar/isovector residual interaction.

![Fig. 1. The isoscalar response is seen to be enhanced for $\hbar\omega > 110$ MeV, compared to the free response, and it diverges for $\hbar\omega > 100$. For the isovector response, where the residual interaction is repulsive, the maximum of the response is shifted about 10 MeV to higher excitation energies. The sum of the energy-weighted response is the same for all three cases shown in the figure (energy-weighted sum rule). For the isoscalar response 25% of this sum is exhausted by excitations less than 10 MeV.](image)

A major problem in shell-model calculation for heavy nuclei is the size of the model space. Even a calculation for semi-closed shell nuclei involves dimensions of the order of several thousands. However, as has been pointed out by Talmi, the generalized seniority (g.s) model provides a valid and powerful truncation scheme. Binding energy systematics and the excitation energy of the $2^+_1$ level indicates that there is only a slight breaking of the g.s. scheme.

The g.s. scheme as it was proposed by Talmi is a generalization of the usual seniority concept to the case of several non-degenerate orbits. In analogy to the treatment of the normal seniority formalism one introduces the operator

$$S_n = \sum_j a_j \frac{1}{2} \sqrt{2j+1} \left( \begin{array}{c} a_{j+1} \end{array} \right)_j$$ \hspace{1cm} (1)$$

together with its adjoint $S_n^+$. Unlike the normal seniority scheme it is however not possible to complete this set of operators with an $S_n = \frac{1}{2} \left( S_n, S_n^+ \right)$ such that the three close under commutation. This means that there is no symmetry group associated with the g.s. scheme and thus that reduction formulas are much more complicated. This has for a long time prohibited any extensive calculations. Also a problem arises in the definition of the basis states since, because of the lack of an underlying symmetry group, not a real g.s. quantum number can be introduced. A g.s. label, \( \nu \), can be introduced by defining a w0 state as

$$|J^0+\nu, J^0=\nu^+\rangle = S^0_\nu |0\rangle$$ \hspace{1cm} (2)$$

where \( K \) is an appropriately chosen normalization factor and \( S^0_\nu \) is defined in Eq. 1. A w0 state can now be defined as

$$|J^0+\nu, J^0=\nu^+\rangle \left( \begin{array}{c} S^0_{\nu+1} \
S^0_{\nu+1} \end{array} \right) |\nu\rangle$$ \hspace{1cm} (3)$$

where the \( \downarrow \) denotes that the state is orthogonalized to the w0 state in the case that \( J=0 \).

Excitation energies and transition rates can be calculated in the g.s. basis provided that the coefficients \( a_j \) are known that enter in the definition of the \( S_n \) pair operator, Eq. 1. The \( a_j \) are determined by requiring that the Hamiltonian does not couple the w0 state with the w0 states, i.e., that in a w0, J=0 basis the ground state is a pure w0 state. To achieve this an iterative procedure has been developed that works as follows. For an arbitrary choice of the coefficients \( a_j \), the lowest energy state in a w0 basis can be written as \( S^0_{\nu+1} |0\rangle \) where the coefficients \( a_j \) that enter on \( S_n \) are different from the \( a_j \) that define \( S_n \). The values \( a_k^* \) which replace \( a_j \) in the next iteration step are taken as a weighted average of \( a_j \) and \( a_j \):
THE TIM ISOTOPES IN THE GENERALIZED SENIORITY SCHEME

O. Scholten

As an application of the generalized seniority scheme, the Sn-isotopes have been calculated. An interaction & surface delta interaction (SDI) has been chosen in which the quadrupole component is enhanced by a factor $F_q$. The strength of the interaction $A_i = 0.18$ and the quadrupole enhancement factor $F_q = 1.9$ have been adjusted as to reproduce the 0-2-4 splittings in the ground state band. The enhancement of the quadrupole component finds its origin probably in core polarization which introduces components in the interaction between like particles that resemble the neutron-proton interaction, which has a strong quadrupole component.

Levels in the odd mass Sn isotopes can be calculated in lowest order in a generalized seniority $\omega$ basis. This calculation should reproduce the lowest levels with spin and parity equal to the single particle (s.p.) states included in the calculation insofar as they are strongly populated in s.p. transfer. In the calculation of the $\omega$ states in the nucleus ($\omega$+) the structure of the $S$-pair state (Ref. 1) is taken the same as what has been determined for the nucleus ($\omega$+2). The single particle energies, listed in Table 1, have been adjusted as to give a best agreement for the low-lying levels in the odd-mass Sn isotopes, and the results of the calculation are compared with experiment in Fig. 1. Since binding energies are not considered, the s.p. energy for the $S_{1/2}$ orbit is arbitrarily normalized to 0.0. The calculated energies of the $3^+$ and $4^+$ levels are compared with experiment in Fig. 2.

A calculation of binding energy systematic, electromagnetic transition rates and single particle transfer amplitudes is in progress.

Fig. 1. Some calculated and experimental energies for Sn isotopes. The levels are labelled with twice the level spin.

Table 1. Determined single particle energies for the Sn isotopes

<table>
<thead>
<tr>
<th>J</th>
<th>$S_{1/2}$</th>
<th>$S_{3/2}$</th>
<th>$I_{1/2}$</th>
<th>$I_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.5</td>
<td>2.0</td>
<td>1.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

1. O. Scholten, contribution to this annual report.
THE N=82 ISO ToNES IN THE GENERALIZED SENIORITY SCHEME

O. Scholten and M. Kruse

An extensive calculation of the N=82 isotones has been done in the shell model. This calculation has resulted in a set of single particle energies and two-body matrix elements which optimally reproduce the observed binding and excitation energies. For members of the N=82 isotope chain above A=140 the calculations become very time-consuming. In a basis which has been truncated on seniority (v ≤ 4), and in which the occupation of the 1½ orbit is restricted, one routinely has to handle matrix dimensions of 1000-2000; the largest dimensions are of the order of 3500.

In order to recover some of the simplicity, the calculations have been repeated in the generalized seniority scheme, and the results of this comparison for excitation energies are reported elsewhere. The agreement between the two calculations for low-lying levels is impressive, especially in view of the fact that the generalized seniority scheme involves matrix dimensions of not more than 10.

We report here on calculation of electromagnetic transition rates in the N=82 isotones. In Fig. 1 the B(E2; {2+} → {0+}) calculated in both the shell model and the generalized seniority scheme are compared to experiment. The shell model values were obtained using Wood-Saxon wave functions and an effective charge $e_{eff}^2 = 1.67$; the calculation in the generalized seniority scheme uses oscillator wave functions and an effective charge $e_{eff}^2 = 1.52$. Both calculations give similar results for the lower-mass members of the isotope chain, the shell model results reproduce the data slightly better for A=144.

Fig. 1. Experimental and computed B(E2) values for the 2+ to 0+ transition in the N=82 isotones. The solid line represents the calculation in the generalized seniority scheme, the dashed line shows the shell model results.

The observed trends in the B(E2) can be easily understood in the generalized seniority scheme: For the lighter isotopes the B(E2) value increases linearly with mass since the B(E2; 2+ → 0+) which connects states that differ in seniority, is proportional to the number of fermion pairs in the valence shell. If Z=64 were a good shell closure, the B(E2) would decrease linearly for 57<Z<64 since increasing the number of protons decreases the number of holes in the 50-64 subshell. Assuming no subshell closure at Z=64, the B(E2) would increase linearly from Z=50 to Z=66. As can be seen from Fig. 1, the B(E2) values display a trend somewhat in between these two extremes; this can be understood in terms of an effective number of fermion pairs as described in Ref. 4.

2. O. Scholten, contribution to this report.
THE ODD-MASS PLATINUM ISOTOPES IN THE IBFA MODEL

O. Scholten and Y. Lee

The Platinum isotopes have always formed interesting test cases for the IBFA model and formed the first evidence for the occurrence of the O(6) symmetry in nuclei. Recently the interest in the Pt isotopes has been revived with the introduction of super-symmetries. Several different reduction schemes for these super-symmetries have been proposed. Super-symmetries offer a simple framework for labeling the states in a nucleus and dictate selection rules for transition. From a microscopic point of view symmetries are often very ad hoc and difficult to understand. For this reason we are calculating the spectra of the odd Pt isotopes in the IBFA model. In this contribution only preliminary results of the calculation for $^{195}\text{Pt}$ will be reported.

In the IBFA model, $^{195}\text{Pt}$ is calculated by coupling the degrees of freedom of a single neutron hole to the systems of bosons describing the even-even core, $^{198}\text{Pt}$. In the discussion of negative parity levels only the single particle levels close to the Fermi surface, i.e. the $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ orbits will be important. In the IBFA calculation the occupancies of these levels was taken as $v^-$ = 0.7, 0.3, 0.2 with relative single particle energies $\epsilon_\frac{1}{2}^+ = 0.0$, 0.35, 0.62 for the $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ orbits respectively. The strength of the boson-boson quadrupole and exchange force were taken as NQ=0.3, BFE=0.5. The value of the $2^+$ parameter appearing in the quadrupole interaction has been taken the same as the value of $3^+$ in the fit to the even Pt isotopes. CNQ = 0.8. In Fig. 1 the calculated energies are compared with experiment and those of a recent super-symmetry calculation. Both calculations agree with the data almost equally well. Also the pattern of the Bi(2) from the ground state $3^+$ to the first 5 excited states is reproduced in both calculations. In spite of all this agreement the overlap of the wave function for the ground state $3^+$ state obtained from the two calculations is only about 0.5, showing that the two calculations differ considerably. In order to distinguish between the two models a systematic calculation of all known odd Pt-isotopes is in progress comparing not only energies but also electromagnetic transitions and single particle transfer amplitudes.

\begin{tabular}{|c|c|c|c|}
\hline
SU6Y  & EXP  & $^{195}\text{Pt}$ & IBFA  \\
\hline
0.0  & -15 & $-15$ & $-15$  \\
0.1  & -13 & $-13$ & $-13$  \\
0.2  & -11 & $-11$ & $-11$  \\
0.3  & -9 & $-9$ & $-9$  \\
0.4  & -7 & $-7$ & $-7$  \\
0.5  & -5 & $-5$ & $-5$  \\
0.6  & -3 & $-3$ & $-3$  \\
0.7  & -1 & $-1$ & $-1$  \\
\hline
\end{tabular}

Fig. 1. The experimental spectrum for $^{195}\text{Pt}$ is compared with the super-symmetry calculation and IBFA calculation. The levels are labelled with twice the spin.

IBFA CALCULATION FOR THE PROMETHIUM ISOTOPES
O. Scholten and T. Ozsello

The Interacting Boson-Fermion Model (IBFM) is used to describe the spectra and electromagnetic properties of odd-even nuclei. In the model, the spectrum of an odd-even nucleus is described by coupling the degrees of freedom of an odd particle to an even-even nucleus which is calculated in the Interacting Boson Model (IBM). We have applied the IBFM to the odd-mass promethium isotopes. Extensive calculations for the neighboring europium isotopes in the framework of IBFM have been published.\(^1\) Comparison of the parameters can yield the proton dependence of the IBFM parameters.

Two separate sets of calculations are done, one for the positive and one for the negative parity levels. To calculate the negative parity levels an \( \pi h_{1/2} \) proton is coupled to the neodymium core for which there exists an IBM calculation.\(^2\) Figure 1 shows the comparison between the calculation and experiment. Three parameters, of which only two are linearly independent, are used to calculate these levels, V5Q, which gives the

![Figure 1](image1.png)

**Fig. 1.** Experimental and calculated negative parity levels of the odd-mass Pr isotopes. The levels are labelled with twice the spin.

![Figure 2](image2.png)

**Fig. 2.** Experimental and calculated positive parity levels of the odd-mass Pr isotopes.
THE ODD-MASS IRIDIUM ISOTOPES IN THE IBFA MODEL
O. Scholten and T. Ossel

In the IBFA model the odd-mass Ir isotopes (2+7F) are calculated by coupling the degrees of freedom of the odd proton to the Pt(2+78) core with the same number of neutrons. In the description of the positive parity levels the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ orbits play a dominant role. The positive parity levels have been calculated before in a super symmetry model where only the coupling of a $\frac{3}{2}^+$ proton hole to an 0(6) core is considered. This model can explain binding energies and B(E2) selection rules quite well. Single particle transfer amplitudes however indicate that already the $\frac{1}{2}^+$ level at 80 keV contains a considerable $\frac{1}{2}^-$ component. For this reason we have started IBFA calculations in which both the $\frac{1}{2}^+$ and the $\frac{3}{2}^+$ orbits are being considered.

In the calculation the parameters in the IBFA Hamiltonian have been adjusted from isotope to isotope so as to give a best agreement for excitation energies. The parameters are listed in Table 1. The calculated excitation energies are compared with experiment in Fig. 1. In this calculation the $\frac{1}{2}^+$ level contains a considerable $\frac{1}{2}^-$ component. A detailed calculation of spectroscopic factors and electromagnetic transition rates is in progress.

Table 1. Parameters used in the Ir calculation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>191</th>
<th>193</th>
<th>195</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{\alpha}$</td>
<td>.20</td>
<td>.20</td>
<td>.60</td>
</tr>
<tr>
<td>$\Gamma_{\beta}$</td>
<td>.20</td>
<td>.60</td>
<td>.80</td>
</tr>
<tr>
<td>$\Gamma_{4/2}$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>$\Gamma_{6/2}$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>$\Gamma_{4/2} - \Gamma_{6/2}$</td>
<td>.1</td>
<td>.1</td>
<td>.25</td>
</tr>
</tbody>
</table>

Fig. 1. Calculated and experimental excitation energies for 181, 191, 193 Ir.


occupational probabilities of the 1/2+ proton orbit, ($\beta_1$), the strength of the exchange force, and ($\Gamma_\alpha$), the strength of the quadrupole-quadrupole interaction. These parameters all vary smoothly as a function of neutron number. Calculations were also done to obtain values for single particle transition amplitudes, B(E2) and B(M1) transitions, along with magnetic and quadrupole moments.

B(E2) VALUES AND LEVEL ENERGIES OF EVEN-MASS
STABLE Dy NUCLEI IN THE IBA MODEL

R. M. Ronningen, O. Scholten, and T. Otsello

It was noted in early Coulomb excitation and inelastic deuteron scattering studies of deformed rare-earth region nuclei that E2 and E3 excitations of vibrational-like \( I^+_2 \) and \( J^+ \) states were, in general, stronger for increasingly more neutron deficient isotopes of a given nucleus. More recent Coulomb excitation studies confirmed this (for discussion and references see Ref. 1). In particular, in the heaviest isotope of each stable even-A Gd, Dy, Er, Tb, and Hf nucleus the \( I^+_2 K=2^+ \) state has the largest B(E2) value to the ground state. The strength and level energy of this "gaama" vibrational state remains fairly constant throughout the region, except for the Tb nucleus. On the other hand, the lowest lying \( I^+_1 K=2^+ \) states (traditionally called beta vibrational states), show some collective (\( -1 \) single particle unit) only in the lightest isotopes. In fact, in the above nuclei the lightest isotope has the most collective \( I^+_1 K=2^+ \) state, except in the case of the Dy nuclei. The level energies generally increase rapidly with increasing neutron number and this particular mode of excitation seems to disappear in the heaviest isotopes.

The stable Dy nuclei should provide interesting tests of nuclear models because their intrinsic shapes range from weakly (N=90) to strongly deformed (N=90), the general behavior of the low-lying vibrational-like states described above, and particularly because of the apparently anomalously small collectivity exhibited by the \( I^+_1 K=2^+ \) vibrational state in \( 154 \) Dy compared to the other lightest nuclei, specifically the isotones \( 154 \) Gd and \( 162 \) Er. We have thus chosen to investigate the Dy nuclei using the IBA model.

The level energies and B(E2) values of the even mass Dy isotopes have been calculated in the IBA-1 model using the Hamiltonian where

\[ H = \frac{\hbar^2}{2I} \left( L^{(1)} + \frac{1}{2} \right) + \frac{\hbar^2}{2I^2} \left( L^{(2)} + \frac{1}{2} \right) \]

and

\[ Q^{(2)} = d^* d + g^{(2)} \frac{4}{25} (d^* d) \]

The four parameters \( c, x, c^*, \) and \( x \) (listed in Table 1) were adjusted for each isotope separately, however, insisting on a smooth dependence of the parameters on mass number. The decrease in \( x \) indicates the transition from an axially symmetric rotor to a Y-unstable shape. In the level energy spectrum shown in Fig. 1 this can be seen from the low-lying \( 2^+ \) level in comparison to the \( 0^+ \) band. The trends of the level energies are well reproduced.

In the IBA model the number of bosons is usually chosen as the number of fermion pairs outside of a closed shell. The choice of closure is ambiguous. A calculation of the effective number of bosons suggests that the number of proton bosons should be taken as \( 8 \) instead of \( 8 \) if \( I=50 \) or \( 1 \) if \( I=64 \) is taken as the closed shell.

The B(E2) values were calculated using the quadrupole operator \( Q^{(2)} \) as the transition operator. We used the value \( x = -1.5 \) for all isotopes. As can be seen in Fig. 2 the trends of B(E2) ratios are well reproduced but the increase of B(E2; \( 2^+ \rightarrow 0^+ \)) is overpredicted. The reason for this could be that also for neutrons the concept of an effective boson number should be introduced. Apparently these numbers should be smaller than the ones used in the present calculations.


Table I. Parameters used in the IBA calculations of the Dy isotopes

<table>
<thead>
<tr>
<th>N</th>
<th>156</th>
<th>158</th>
<th>160</th>
<th>162</th>
<th>164</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>156</td>
<td>158</td>
<td>160</td>
<td>162</td>
<td>164</td>
</tr>
<tr>
<td>B</td>
<td>0.295</td>
<td>0.266</td>
<td>0.133</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>c</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0025</td>
</tr>
<tr>
<td>c'</td>
<td>-0.0413</td>
<td>-0.0518</td>
<td>-0.0585</td>
<td>-0.0585</td>
<td>0.0585</td>
</tr>
<tr>
<td>x</td>
<td>-2.50</td>
<td>-2.00</td>
<td>-1.50</td>
<td>-1.33</td>
<td>-1.33</td>
</tr>
</tbody>
</table>
Fig. 2. Experimental and calculated $B(E2)$ values for the stable even mass $^{156,160,162,164}$ Dy isotopes.
ELASTIC MAGNETIC ELECTRON SCATTERING FROM g SHELL AND d SHELL NUCLEI

S. Reich, B. A. Brown and B. M. Wiendenhal

We have calculated the form factors for elastic magnetic electron scattering from all stable nuclei below A = 40 which have non zero-ground state spins. These form factors provide very detailed and specific information about the structure of the nuclear states which are probed. Comparisons of model predictions with experimental data thus yield important critiques of the theoretical descriptions of these states. We have used such comparisons to test the shell model wave functions recently derived for the p-shell and sd-shell regions (Ref. 1, Ref. 2). These wave functions result from calculations which assume that the two-body matrix elements of the effective model Hamiltonian decrease with increasing A as a power of (1/A).

We have pursued these comparisons of theoretical with experimental form factors beyond the arena of tests of specific shell model configuration mixing in order also to investigate the role of higher-order corrections to the basic elements of the shell-model ansatz. Specifically, we have tested the degree to which changes in the values of the magnetic g-factors in the conventional expressions of the magnetic operators affect the predicted form factors. We have also studied the dependence of the calculated form factors upon the radial dimensions of the single-particle wave functions which form the basis of the shell model.

The "single-particle" nuclei, those which correspond to one nucleon more or less than a "magic" closed shell nucleus such as 16O or 40Ca have particularly simple features in the conventional shell-model representation. The wave functions for their ground states are just single-particle orbits and are hence independent of the interference effects of the configuration mixing induced by the residual two-body Hamiltonian. Within the confines of the usual shell-model assumptions, then, deviations between theory and experiment for the single-particle nuclei are immediately attributable to the sort of higher-order effects mentioned above.

Elastic magnetic form factors calculated for 170 are shown (solid lines) in Fig. 1 in comparison with the experimental data (Ref. 3). The model wave function employed in the calculation is that of a 0+ 170 neutron. We use a representation (Ref. 4) which allows us to display the conventional magnetic moment along with the electron scattering data. The left-hand panels of Fig. 1 show predictions based on harmonic oscillator radial dependence while the right-hand panels show the corresponding results based on a Woods-Saxon potential radial dependence. In both instances the scales of the radial wave functions are set originally to reproduce the measured charge radius of 170. It can be seen that in all instances there is little substantive difference between the harmonic oscillator and Woods-Saxon results.

The top panels show the results which are obtained by giving the g-factors which enter into the conventional formulations of the operators for the various magnetic multipoles the value measured for the free neutron. The theoretical results reproduce the measured magnetic dipole moment of 170, a fact long well known. They deviate from the experimental scattering data in the regions of momentum transfer q around 1.0 fm\(^{-1}\) (theory larger than experiment) and 2.5 fm\(^{-1}\) and greater (theory smaller than experiment). The contributions of the various magnetic multipoles to the theoretical form factors are indicated in Fig. 1 by the dotted lines (M1), the dashed lines (M3) and the dashed-dotted lines (M5). The sums of the squares of the contributions give the squares of the total form factor values. The lines show the absolute values of these individual multipole amplitudes. We see from these decompositions that the M3 contribution dominates in the 1.0 fm\(^{-1}\)
region and that the MS contribution dominates at beyond 3.0 fm⁻¹. The middle panels of Fig. 1 show the results of using a value of $g_3$ in the M3 operator which is 0.60 times the free neutron value. This has the simple consequence of reducing the (M3) dominated form factor in the region of 1.0 fm⁻¹ to approximately the measured magnitudes. This change leaves both the lower $q$ and higher $q$ portions of the calculated form factors essentially unchanged. No such simple scaling of $g$-factors can rectify the disagreement between theory and experiment at the highest $q$ values, however. One simple alteration to the conventional values of the basic shell-model parameters can remove most of the high $q$ discrepancy. This alteration is to use a $0g_{9/2}$ single-particle wave function whose rms radial size is 0.95 times the "conventional" value. The "conventional" value is determined by reproducing the measured rms charge radius under the assumption that all occupied single-particle states of the shell model (including both "core" and "active" orbits) are generated from a single parametrization of the single-particle potential. The bottom panels of Fig. 1 show the effects of using this reduced radius for the $0g_{9/2}$ wave function in conjunction with the quenched value of $g_3$ for the M3 multipole. Both the harmonic-oscillator and Woods-Saxon versions of this prescription of the model form factor agree well, by our standards, with the measured moment and form factor data. We are left with questions about the significance of these alterations of "conventional" parameters which have the effect of improving agreement between the predictions of the single-particle model and experiment. Are the adjustments needed for $^{17}$O merely ad hoc compensations for an overly simple model and in themselves devoid of physical significance? Or, do they provide a concise summary of the observable consequences of the single-major-shell model approximation which can be related to theoretical estimates of the higher-order corrections to this conventional picture of nuclear structure?

The obvious next step in this inquiry is to examine the case of $^{39}$K, the nucleus which corresponds to a single vacancy ($0g_{9/2}$) in the sd-shell. In Figs. 2 and 3 we pursue the same issues of $g$-factor renormalization and single-particle wave function radius. In Fig. 2a we show the predictions of the "standard model", in which the $0g_{9/2}$ proton is assumed to have its free-space $g$-factor and an harmonic-oscillator radial wave function with a radius which is consistent with the measured charge radius of the nucleus. The experimental data is from Ref. 3 [circles—(Amsterdam data) and squares—(States data)]. The case of $^{39}$K has an extra degree of complexity relative to $^{17}$O in that the charge of the proton contributes a current term which is absent to lowest order in the case of the single neutron of $^{17}$O. The bottom two panels of Fig. 2 show the individual spin and orbital contributions to the total form factor presented in the top left panel. We plot absolute values, and the spin contribution shown in the bottom right panel is negative at $q$-values of less than 0.7 fm⁻¹. The well-known result that the small magnetic moment of $^{39}$K results from a cancellation of the spin and orbital contributions is evident from Fig. 2 as is the fact that the spin contribution dominates the scattering at large momentum transfers.

![Fig. 2. Comparison of experimental and theoretical elastic magnetic scattering form factors and magnetic moments for $^{39}$K. See text for details.](image)
Fig. 3. Comparison of experimental and theoretical elastic magnetic scattering form factors and magnetic moments for $^{19}K$. See text for details.

In Fig. 3 we illustrate the effects of altering the radius of the single particle wave function and of making more detailed alterations in the M1 g-factor renormalizations. As should be expected, the use of a smaller radius parameter causes the calculated values to increase at large momentum transfers and, hence, to agree better with the data. Either the spin or the orbital term can be altered to bring the calculated and measured dipole moments into agreement with each other. Not all such choices are consistent with the scattering data, as is shown by Fig. 3b, where the results of increasing the orbital g-factor by a factor of 1.15 and leaving the spin g-factor unchanged are compared with experiment. Fig. 3c shows the results of a combination of renormalized M1 spin and orbital g-factors which agrees reasonably well with both moment and scattering data. Fig. 3d shows the same calculation as in Fig. 3b, but in this case Woods-Saxon rather than harmonic oscillator radial wave functions have been used. The best agreement with experiment is obtained by this last calculation.

Similar calculations for all magnetic elastic form factors for the p and s shells nuclei have been carried out and compared to experiment where available (Ref 5). Away from the closed shells the additional effects of configuration mixing are essential in interpreting the comparison between experiment and theory (Ref 6).
INELASTIC ELECTRON SCATTERING FROM $^{27}$Al

R. Radhí, B.A. Brown and B.H. Wildenthal

Recent measurements by Ryan and co-workers$^2$ of the inelastic scattering cross sections of electrons from $^{27}$Al sample extensive ranges of excitation energy and momentum transfer q and are characterized by exceptionally good energy resolution and freedom from background contamination. They hence provide a rich field within which to test theories of nuclear structure with precision and thoroughness. We compare the results of this experiment with predictions of longitudinal and transverse electron scattering form factors derived from wave functions obtained in a new shell-model calculation for the positive-parity states of $^{27}$Al. This new calculation employs the full $0g_{9/2}$-like $0g_{7/2}$ basis space and an effective Hamiltonian which gives an accounting of the positive-parity level structures of the entire sd-shell region. From this comparison of experiment with theory we attempt to learn both about the general characteristics of the conventional shell-model vis-à-vis electron scattering phenomena and about the specific model wave functions used to generate the theoretical form factors.

In the first instance, we look for systematic deviations between model predictions and experiment which can be remedied by alterations in the formulations of the electromagnetic operators. These alterations can be thought of as compensations for the restrictions of the model to a single oscillator shell and to purely nucleonic degrees of freedom. At their simplest, they would manifest themselves as shell-wide renormalizations of the strengths of the operators. Extensive knowledge on this aspect of the relationship between the conventional shell model and experimental data is available from comparisons of shell-model predictions and measured values of $M_1$, $E_2$ and Gamow-Teller matrix elements.

Electron scattering measurements make it possible to extend the range of such comparisons to higher multipoles. In practice, these are not observed in decay experiments. Of equal or greater importance, electron scattering also allows the momentum-transfer dependence of excitation processes to be examined. It is possible that the conventional shell-model approximations have consequences which are revealed as discrepancies between the predicted variations of multipole strength with momentum transfer q and the observed variations. Such discrepancies could be characterized as the need to employ q-dependent renormalizations to the operators.

In the second instance, we wish to test the specific assumptions about shell-model space and interaction which determine the actual wave functions used to generate the theoretical form factors. This we do by ascertaining whether the observed excitation strengths of the various multipoles which can participate in the scattering processes to the different final states are correctly predicted. The theoretical strengths are functions of the different mixings of the ad-shell configurations into the corresponding model states which occur in the diagonalizations of the Hamiltonian. These comparisons involved the relative strengths of the different allowed multipoles within a given transition, the relative strengths of a given multipole from transition to transition, and the overall normalizations of the transition strengths.

Obviously, the tests of the general characteristics of the shell-model approximation and of the specific individual wave functions are not completely disjoint. It is necessary to analyze an extensive set of related data with an internally consistent set of predictions before meaningful conclusions can be drawn independently about both of these aspects of nuclear structure theory. The $^{27}$Al data of Ref. 2 and the wave functions of Ref. 1 provide necessary ingredients for such an analysis.

We show in Figs. 1 and 2, as examples of how theory and experiment compare, the form factors of the first and second 7/2$^+$ stages of $^{27}$Al, at excitation energies of 2.21 and 4.58 MeV respectively. The longitudinal (and total at 90°) scattering to the lower state is observed to be dominated by the E2 multipole. The total scattering at 90° to the higher state (essentially all longitudinal, but a formal separation was not experimentally effected) is observed to be dominated by the E4 multipole. This observed qualitative change in dominance from E2 to E4 is in complete accord with the predictions for these two states. The experimental strengths with which the two states are excited (the maximum values of the form factors) are also accurately reproduced by the theoretical curves. Finally, the observed dependence of the excitation strengths upon the momentum transfer q are accurately reproduced over the range 0.5 to 6 fm$^{-1}$.

The theoretical form factors plotted in Figs. 1 and 2 incorporate single-particle wave functions of the harmonic-oscillator form, with the size parameter set to produce agreement with the measured rms charge radius, and simple “operator renormalizations” of the “effective charge” 1.59. The values for these renormalizations are those obtained in surveys of B(E2) values and E2 and E4 electron scattering from double-even nuclei in the sd-shell. The E2 operator is effectively altered such that the model protons are assumed to have charges 1.35e and the model neutrons 0.35e. The E4 operator is
Fig. 1. Inelastic electron scattering form factors for the first 7/2 state of 28Al, which occurs at an experimental excitation energy of 2.21 MeV. The top panel shows the separated longitudinal form factor, the middle panel the separated transverse form factor, and the bottom panel the total (unseparated) form factor at 90°. The lines show the full theoretical form factors in each case. The "+" and "x" symbols in the longitudinal panel show the E2 and E4 components, respectively. In the transverse panel, the M1, E2, M3, E4 and M5 components are shown by the "+", "x", "x", "x" and "triangle" symbols, respectively. In the 90° plot, the solid lines show the total, transverse plus longitudinal, form factor while the dashed-dotted line shows the transverse component alone, multiplied by the angular factor.

Fig. 2. Inelastic electron scattering form factors for the second 7/2 state of 28Al, which occurs at an experimental excitation energy of 4.58 MeV. The conventions of the presentation are the same as those used in Fig. 1.

We see from Figs. 1 and 2 that the wave functions correctly select out the dominant excitation multipoles, that the conventional effective charge values correctly normalize the strengths of these transitions and that the total model transition densities comprised of the valence sd-shell terms plus the Tassie-model core-polarization corrections successfully reproduce the form factors over the entire range of measured momentum transfer. We note also that the measured B(E2) of the electromagnetic decay between the first 7/2+ and the ground state is correctly reproduced with this same calculation.

The same two states are also populated by the transverse component of the electron
scattering process. The spins of the initial and final states are such that several multipoles can participate in the transitions. The envelopes of the theoretical sum of these multipole components agree reasonably well with the data for the two states. In addition, the measured $B(M1)$ for the lower state is reproduced by the theory. The theoretical form factors incorporate the free-nucleon values of the charges and $g$-factors. The complexity of these particular data do not permit enough discrimination to determine if alterations to these normalizations are needed. Additional transverse measurements would allow more definitive conclusions on the apparent $g$-independence of the shell-model agreement with experiment.

We reported earlier on the computation of energy levels in the N=82 isotope chain of nuclei. We have used the wave functions obtained in this work to determine spectroscopic factors and electromagnetic transition rates in the N=82 nuclei. We expect that these transition operators will provide a sensitive test of our wave functions. In addition the spectroscopic factors can provide some information on the nature of the subshell closure at $^{184}$Os.

Fig. 1. (a) Experimental pickup strength from the 5/2$^+$ orbit compared to the shell model prediction (solid line); (b) Same as (a), but for the stripping strength into the 5/2$^+$ orbit.

Figure 1 shows one example for the computation of spectroscopic factors: shown is the stripping strength into the 5/2$^+$ orbit as well as the pickup strength from the 5/2$^+$ orbit. Note that the spectroscopic factor varies smoothly across the subshell closure, a trend which is also displayed in the data; this seems to indicate that the closure at Z=64 is a very soft one. This conclusion is supported by the pickup and stripping strengths for the remaining orbits.

Figure 2 shows the B(E2) for the 4$^+$ to 2$^+$, and the 2$^+$ to 0$^+$ E2 transitions. The B(E2) values were computed using Woods-Saxon radial wave functions and an effective charge of $e_\text{eff}^2=1.67$. We note that the strong 2 to 0 transitions are in general well reproduced, whereas there are larger discrepancies in the weaker 4 to 2 transitions which sample small components of the nuclear wave functions.

A systematic study of electromagnetic transitions in the N=82 nuclei is currently in progress. We hope to use this information to improve our wave functions, and to the understanding of the electromagnetic properties of the nuclei in this region, and provide a guide for the interpretation of ongoing and future experiments involving electromagnetic transitions in these nuclei.

SPIN-TENSOR ANALYSIS OF NUCLEAR EFFECTIVE
INTERACTIONS IN THE 1s-0d SHELL

B. A. Brown, W. A. Richter* and B. H. Wildenthal

* Permanent address: Physics Dept., University of
Stellenbosch, Republic of South Africa.

The calculation of the shell-model effective interaction starting from the free nucleon-nucleon
interaction is one of the most important
problems in nuclear physics. Considerable
progress has been made in calculating the finite-
nuclei reaction or G matrix. However, the
renormalization of the bare G matrix required by
the inevitably use of a truncated shell-model
space is troubled by many uncertainties. In this
work we have compared the calculated effective
two-body matrix elements based on the nucleon-
nucleon interaction with the empirical matrix
elements found from fits to binding energy data in
the sd shell. The study is motivated in part by
the need for an effective interactions in heavier
mass regions of a quality similar to those
available for the 1s-0d shell. We hope that by
thoroughly analyzing the features of the sd-shell
interactions, the essential virtues of the
empirical result can be translated to higher
(larger) shell-model spaces.

The important aspects of the interaction can be more readily discerned by
transforming the two-body j-j coupled matrix elements, which is the representation generally
needed for shell-model calculations, to the L-S coupling scheme, following this by a spin-tensor
decomposition. The essence of the spin-tensor
decomposition method employed has been described
by Khoa2 and Toro.3 The two-body interaction can be written in the form

\[ Y = \sum_k V_k - \sum_k U_k \cdot S_k, \]

where the operators U and S are irreducible
tensors of rank k in space and spin coordinates,
respectively. In order to obtain the L-S coupled
matrix elements, a j-j to L-S coupling
transformation is applied:

\[
\begin{align*}
&\langle ab|LSJT|k \rangle = \sum_{J} \sum_{J'} \langle ab|J^+J^1\rangle \langle J^1J|L'S'J'\rangle \\
&(2k+1) \langle L S J J' |\{\pm 1\}^0 |L' S' J' \rangle \\
&\left(\begin{array}{c}
(1+\delta_{ab})/2 \\
(1-\delta_{ab})/2 \\
\end{array}\right)^{1/2} \\
&\left(\begin{array}{c}
J_a+J_b+J_{ab} \\
J_a-J_b+J_{ab} \\
\end{array}\right) \\
&\{(2J_a+1)(2J_b+1)(2J_{ab}+1)(2J_{ab}+1)^2\}^{1/2}
\end{align*}
\]

where a = (n_a, 1/2), J_a = (n_a_i, 1/2), etc., in
the matrix elements, and both two-body matrix
elements are normalized and antisymmetrized. The
nomenclature for the separated components is explained in the following table:

<table>
<thead>
<tr>
<th>k</th>
<th>S</th>
<th>S'</th>
<th>Spin-tensor components</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>L = central</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>AL0 = antisymmetric</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>spin-orbit</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>LS = spin-orbit</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>T = tensor</td>
</tr>
</tbody>
</table>

For the the 1s-0d shell the empirical
effective matrix elements we have compared are
those of Wildenthal (W),4 Chung-Wildenthal-
particle (CWP), Chung-Wildenthal-hole (CWH),5,6
and Freedom-Wildenthal (FW).7 The W matrix elements have a mass dependence of (A/10)^{-0.3}
and our comparisons are made for A=18. These are compared
with the calculated bare (BK) and renormalized
(RK) G-matrix elements of Kuo based in the
Hamada-Johnston potential,8 as well as with the
more recent results of Shurpin, Kuo and Strottman
(SKS) based on the Reid soft-core (SKSR) and the
Paris (SKSF) potentials.9 The renormalized Kuo
calculations include only the lowest second-order
corrections, whereas the SKS calculations use the
folded-diagram technique and includes 3rd and 4th
order correction terms (we use the "C4" matrix
elements of Ref. 9).

In the comparisons it should be remembered
that the BK matrix elements were used as the
starting parameters for the empirical matrix
elements. For the PW and CW matrix elements only
about 10 linear combinations of parameters were
well determined from the experimental binding
energies, whereas for the W interaction, 17
parameters were allowed to vary. Thus, we find in
the comparison that the PW and CW matrix elements
which were not well determined remain close to
the BK values. The W matrix elements are such more,
independent of their origins. In order to get a
completely independent set, as well as to obtain
an estimate of the errors on the empirical matrix
elements, we have repeated the fit based on 440
binding and excitation energy data which was used to obtain the W interaction, and allowed all 63 two-body matrix elements and three single-particle matrix elements to vary (the BW matrix elements). The values of the W and BW matrix elements are essentially the same.

Our results are illustrated by the comparison of the central interaction components shown in Fig. 1. The 20 matrix elements are labeled by numbers (n) corresponding to the quantum numbers $n_1 n_2 n_3 n_4 l s t$

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The figure is divided into three panels. In each of the panels the BW matrix elements are shown by circles with error bars connected by a solid line. In the top panel, BW is compared with the PW (plus signs) and CWP (small dots connected by dashed lines) matrix elements. In the middle panel, BW is compared to the SKS matrix elements obtained from the Reid soft-core potential (plus signs) (SKSR) and the Paris potential (SKSP) (small dots connected by dashed lines). In the bottom panel, BW is compared to the NK (plus signs) and BK (small dots connected by dashed lines) matrix elements of Kuo.

In this comparison we can note the following: All empirical matrix elements are well determined except for those with $(0,0)$ which have relatively large error bars. The differences between the empirical BW, PW and CWP matrix elements are surprisingly small given the superiority of BW (W) over PW and CWP in reproducing ad shell binding energies and excitation energies. The differences between the G-matrix elements obtained from the Reid (SKSR) and Paris (SKSP) potentials are similar compared to the differences between the original Kuo (BK) and the new (SKSR) results. This illustrates that the major uncertainty lies in the renormalization calculations and probably not in the nucleon-nucleon interaction. In the Kuo calculation the renormalization is important to obtain improved agreement with BW. The renormalization is not as large in the more complete SKS calculation.

Unfortunately the agreement with BW becomes worse. The repulsive $(0,0)$ and $(1,1)$ SKS matrix elements are in reasonable agreement with BW, but the important attractive $(0,0)$ and $(1,0)$ SKS matrix elements are too weak compared to BW.

Similar comparisons have been made with the other components of the interaction. From these comparisons we hope to obtain an improved method of calculating reliable effective interactions to be used in nuclear-stimulated calculations.

References

Recently the results of a series of \((p,\gamma)\) experiments on a number of nuclei have been reported by the Indiana group (Vigrod et al.)\(^1\). They have pointed out that this reaction populates selective high spin states which can be correlated in many instances with those identified previously in gamma decay and particle transfer experiments.\(^2\) In addition, they have found evidence from analyzing power measurements that this reaction involves a relatively simple two-nucleon pion production process.\(^2\) For the Ca isotopes and the \(N=28\) isotones, the \((p,\gamma)\) cross sections for populating discrete low-lying final states have a strong dependence on \(N\) and \(Z\). The cross sections integrated over the strong states below 5 MeV excitation energy in the final nucleus for 206 MeV protons and a pion emission angle of 30° are in units of nb/\(\text{sr}\) 10.6±1.3 for \(^{43}\text{Ti}\), 8.8±2.0 for \(^{45}\text{Ti}\), 53.2±1.4 for \(^{47}\text{Ti}\), 18.0±3.0 for \(^{51}\text{Cr}\) and 9.9±1.5 for \(^{53}\text{Fe}\)\(^3\).

The final states most strongly populated in the \(\alpha^2\text{Ge}(p,\gamma)^{40}\text{Ca}\) reaction comprise a doublet,\(^4\) the higher member of which is consistent in energy with the \(19/2^+\) state at 6.4 MeV excitation energy observed in a gamma decay experiment.\(^5\) However, the lower peak, at 4.0 MeV, cannot be associated with any high-spin state previously identified. Moreover, in a recent experiment with 165 MeV protons,\(^6\) the pions from these two states have been observed to have quite different angular distributions. The cross section of the 4.4 MeV peak falls off by about a factor of two over the angular range of 30–130° while the cross section of the 3.9 MeV peak falls by about an order of magnitude in the same angular range.

We have found that all of the above features of the relative cross sections for these nuclei can be qualitatively understood within the contexts of the \((1f_{7/2})^2\) shell model and some general assumptions about the reaction mechanism in the plane-wave Born approximation limit. From the basic Feynman diagrams for this reaction shown in Fig. 1 it can be seen that the reaction involves three nucleons. The box at the outgoing pion line in Fig. 1 can be seen to contain the \(s\)-wave and/or \(p\)-wave vertex.\(^6\) As the proton energy increases, the \(p\)-wave process with a delta-isobar intermediate state excitation becomes dominant.\(^6,7\) The general nuclear structure amplitude for this reaction can be expressed in terms of matrix elements of creation and annihilation operators. In order to separate the spatial and spin degrees of freedom it is useful to express these amplitudes in LS coupling

\[
\begin{align*}
\langle l_s l_p | J_{\pi} \rangle &= \left(\frac{\alpha}{2}\right)^{J_{\pi}+\frac{1}{2}} \cos^{L_{\pi}} \sin^{\frac{1}{2}J_{\pi}} \\
\langle J_{\pi} | J_{\pi} \rangle &= \left(\frac{\alpha}{2}\right)^{J_{\pi}+\frac{1}{2}} \cos^{L_{\pi}} \sin^{\frac{1}{2}J_{\pi}} \\
\langle J_{\pi} | J_{\pi} \rangle &= \left(\frac{\alpha}{2}\right)^{J_{\pi}+\frac{1}{2}} \cos^{L_{\pi}} \sin^{\frac{1}{2}J_{\pi}} \\
\end{align*}
\]

where \(\alpha = R(q_z, z) R(q_{\pi}, r) R(q_{\pi}, z)\) (L/I) stands for the quantum numbers of the initial and final states and \(\alpha\) stands for the set of quantum numbers \((\pi, \lambda)\) of the shell-model orbits. \((J/I)\) is the normalized \(3\)-\(1\) coefficient. In the cases of interest here \(\alpha\) is \(\pi = 1, \lambda = 1\) for \(J_{\pi} = 0\) and hence \(J = J_{\pi}\) for the total angular momentum transfer. In our \((J/I)\) case and in the zero-range approximation used below the total spin transfer can only be \(S=1/2\) which can be understood by first coupling the two protons to spin \(S = 1/2\). Since the two protons are in the same orbit, \(S_{\pi} = 0\) and thus \(S = 1/2\). In order to conserve parity in the zero-range approximation, L must be odd, and thus each final state spin has a unique total \(L\) transfer given by \(L = J_{\pi} + 1/2\). These amplitudes have been calculated with the wave functions obtained with the \(^{12}\text{C}\) interaction of Ref. 8. A harmonic oscillator potential with \(\hbar\omega = 10.5\) MeV was used for the bound-state radial wave function \(R(r)\).

Since the major part of this momenta has to be carried away by the interaction like in Fig. 1, the interaction is of short range and the zero-
range approximation should be adequate. A plane wave calculation for the incoming proton and outgoing pion then leads to the following expression for the cross section

\[
\frac{d\sigma}{d\Omega} = \frac{4t_{p}^2}{\sin^2 \theta} \int \frac{d\Omega_{\pi}}{4\pi} \frac{\Gamma_{p\pi}}{2\pi} \left( \frac{\Gamma_{p\pi}}{\Gamma_{\pi}} \right)^2 \times \frac{1}{2} \frac{1}{\Gamma_{\pi}} \left( \frac{\Gamma_{\pi}}{\Gamma_{p\pi}} \right) \frac{1}{2} \frac{1}{2} \frac{1}{2}
\]

For our one-orbit calculation it can be shown that \( A(S_{\pi}=1) = (3)^{1/2} A(S_{\pi}=0) \) and hence

the \( S \) dependence enters in our case just as an overall scale factor (independent of angle) in the cross sections. (The results presented here were obtained with \( A(S_{\pi}=1) = 1 \) and \( A(S_{\pi}=0) = 0 \).)

The reaction will be peaked at the nuclear surface because of the strong pion absorption. This can be taken into account in an approximate fashion by using a lower cutoff in the above radial integral. The angular distributions are in fact rather sensitive to this cut-off. We have chosen the value of 3.5 fm (about 1.1 \( A^{1/3} \)) which gives the best reproduction of the experimental data angular distributions. The momentum transfer as a function of angle is \( q = |\mathbf{p}_{\text{eff}} - \mathbf{p}_{\text{eff}}| \)

where the effective momenta take into account the Coulomb effect and the nuclear surface. The most important aspect of the kinematics is the large momentum mismatch. For \( \varepsilon_{p} = 200 \text{ MeV} \), the angular momenta transfer for the pions outgoing at \( 0^\circ \) is \( \Delta l = l \), \( \Delta \not{\tau} = 1 \) for \( ^{40}\text{Ca} \) (and is as large as 17 for backward angles). Hence, this reaction excites high spin states (high orbital angular momentum states) almost exclusively.

The cross sections (ignoring the relatively small \( Q \)-value dependence in the radial integral) for a given \( L \) transfer (\( L=3 \)) are proportional to the probability of stripping two protons into an \( f_{7/2}^2 \) orbit with \( \Gamma_{pp} \) multiplied by the probability of picking up a neutron from an \( f_{7/2}^1 \) orbit. For the \( f_{7/2}^2 \) seniority-zero ground configurations of the Ca isotopes and \( N=28 \) isotones it can be shown that this spectroscopic sum rule is proportional to \( (N-28)/(28-2) \) for each \( L=J \) for \( N \) and \( Z \) of the target. For the cases of interest these numbers are proportional to 12 for \(^{43}\text{Ti} \), 24 for \(^{45}\text{Ti} \), 48 for \(^{47}\text{Ti} \), 96 for \(^{51}\text{Cr} \), and 192 for \(^{53}\text{Cr} \). Except for \(^{45}\text{Ti} \), these values are closely proportional to the experimental cross sections noted above.

The calculated spectra for \( \varepsilon_{p} = 200 \text{ MeV} \) and pions at \( 30^\circ \) are shown in Fig. 2. The agreement with experiment, as excellent. Several details of the spectra which have not been previously understood come out of our calculations. In the theoretical \(^{40}\text{Ti} \) spectrum there are only two strong states one \( 1\text{D}^2 \) and one \( 1\text{D}^2 \). They agree in energy with the two states observed experimentally within a few hundred keV. Since they have different \( L \) transfer values, the angular distributions are very different and at 100\(^\circ \) the 1\text{D}^2 \( \text{L}=9 \) state dominates the spectra. The 1\text{D}^2 \( \text{L}=9 \) state is much stronger than the 1\text{D}^2 \( \text{L}=9 \) state because
the latter is unfavor ed in the J J to LS transformation (this same difference between 19/2- and 17/2- also occurs, for the same reason, in three nucleon transfer. 9) Whereas the lowest 15/2- (L=7) state is strong in 133I, it is the second 15/2- state which is strong in 135I. This is due to the change in going from particle- particle to the particle-hole structure in the wave functions. This feature of the 15/2- states in 135I is confirmed by comparisons with gamma decay data, 5 in which only the yrast 15/2- state is seen, and by comparison with recent 137I(d,x) data, 10 in which both 15/2- states (and other high-spin states) are seen with relative strengths in agreement with \( R_{15/2-} \) calculations. Finally we note the fraction of the total L=7 and L=9 strength concentrated in the low-lying strong states for the five final nuclei (A) of interest are 92\% (13I), 62\% (135I), 89\% (137I), 83\% (151I) and 91\% (153I). This increased fragmentation in 135I explains most of the deviation in this case from the trends expected from the sue rules noted above (the remaining strength in 137I is fragmented over many levels but mainly in the 7+3/2, 3+ = 15/2- and 19/2- states around 10 MeV in excitation).

The features of the spectra are dominated by an "X-window" created on the lower side by the momentum mismatch in the reaction and on the higher side by the maximum transfer \( (L_{\text{max}} = 9) \) available for three particles in the fp shell. It will thus be interesting to pursue experimentally the \((p, x)\) reaction on \(^{82}\text{Sr}\) \((L_{\text{max}} = 12)\) and on \(^{148}\text{Sm}\) \((L_{\text{max}} = 15)\). In \(^{82}\text{Sr}\) we expect a cluster of three closely spaced states with L=12, 10 and 8 around 4 MeV in excitation. (More angular momentum transfer can be achieved by putting the proton into the higher shells and this process may be part of the continuum seen above the discrete states). The success of our simple calculations should encourage progress in the development of distorted wave calculations. Indeed, because of the selectivity of this reaction, a great deal may be learned from such calculations about the the interactions of pions in nuclei.

References
A CALCULATION OF THE CONTINUUM IN \((p,\pi^-)\) SCATTERING
O. Scholten and H. Toki

Recently the interest in the \((p,\pi^-)\) reaction has been growing. It has been demonstrated to be a valuable tool in spectroscopic studies. In order to make full use of it as a spectroscopic tool, the origin of the continuum has to be understood. At 200 MeV the \((p,\pi^-)\) spectrum has been measured up to an excitation energy of about 20 MeV for different targets. The continuum turns out to have a rather peculiar structure, it rises linearly with excitation energy starting at about \(E_x = 5\) MeV.

We have calculated the continuum in a simple fermi-gas model assuming that the process goes via a \(5\) intermediate state. For forward angle scattering the cross section can be written as

\[
\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2} \int \frac{d^3p_n}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} (2\pi)^4 \delta^4(p_n + p - q) \frac{g^2}{(\omega - E_p + i\Gamma/2)^2} \frac{1}{(8\pi)^2}
\]

We have made here the approximation that the momentum transfer \(q\) is large, such that the interaction can be approximated by a \(5\) function. The integration limits are determined by the fact that \(k_n\) should lie inside the fermi sea while \(k_p\) and \(k_{p'}\) have to be above. The integral over the particle hole states, \(E_n\) and \(E_{p'}\) give rise to the particle hole response of a fermi gas which can be written as the imaginary part of the Lindhard function. After this substitution only a two dimensional integral remains to be done numerically. In Fig. 1 the calculated continuum is obtained. The next stage of the calculations is to reproduce the absolute cross section. Preliminary calculations look promising.

1. IUCF newsletter 31, October 1982.
2. B.A. Brown, O. Scholten and H. Toki, to be published.

**Fig. 1.** Calculated continuum cross section for \((p,\pi^-)\) at \(E_p = 200\) MeV