DICKE COHERENT STATE AS A SIMPLE MODEL OF STRUCTURE DETERMINED BY CONTINUUM

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In all realistic quantum systems, the excited states are in fact unstable. An interaction with the outside world leads to the finite lifetime τ and energy uncertainty (decay width) $, \sim \hbar/\tau$ of any state, except for the ground one (even that one is unstable in radioactive nuclei). A standard quantum-mechanical description of unstable states introduces their complex energies

$$\mathcal{E} = E - \frac{i}{2}, \tag{1}$$

which characterize the time development of the wave function $\sim \exp[(-iE - , /2)t/\hbar]$, or decrease of the probability $\sim \exp(-, t/\hbar)$.

Experimentally the unstable intermediate states are studied in the processes of their excitation and deexcitation. As long as typical widths are small compared to level spacings, the states are seen as separated resonances in the cross sections of corresponding reactions. As excitation energy and level density increase, the pattern becomes more complicated due to the overlap of the resonances. In the overlap region, as a rule, the structure of individual states is so complicated that the statistical approach seems to be the only available method of description. The average over the resonances reveals the smooth component of the reaction cross section, which is related to motion of participants in the mean optical (absorptive) potential, and fluctuations caused by nearly random phases of overlapped states. Being observed in the continuum processes, this picture is considered usually a part of scattering theory.

However, one can complement this view from the "outside" with a study of the "inside" structure of unstable states. The intrinsic properties can be noticeably influenced by the decreasing lifetime and the overlap of resonances [1]. At some degree of overlap, the coupling of intrinsic states through a common decay channel leads to a sharp restructuring of the resonance spectrum. This "phase transition" singles out a specific coherent combination of states which accumulates a large fraction of the total width of all coupled states. The underlying physics is similar to that of giant resonances or to the Dicke state in coherent optics [2].

The so-called effective nonhermitian hamiltonian obtained by a projection of the full Schrodinger equation onto subspace of intrinsic states can serve as a convenient tool for studying internal dynamics in the unstable system. The parameters of the hamiltonian directly describe the coupling with and through the continuum and, at the same time, the scattering matrix, which has the complex energies (1) of decaying states as its poles. The construction guarantees the unitarity properties and therefore is not simply equivalent to a coupling with a heat bath leading to irreversible relaxation. The practical advantage of using the effective hamiltonian is a possibility to apply standard methods developed for the discrete spectrum.

The relevant physical coupling parameter is the ratio of typical widths of the resonances to their energy spacing,

$$\kappa = \frac{\langle , \rangle}{D}. \tag{2}$$

At $\kappa \ll 1$, the spectrum consists of narrow well isolated resonances. In the opposite case of strong coupling, $\kappa \gg 1$, we would expect the original states $|n\rangle$ to overlap. However, the specific structure of the effective hamiltonian , which is dictated by the requirements of the unitarity of the scattering matrix, shows that a number of states, equal to a number k of open channels, accumulates the lion's share of the total width of all intrinsic states. The remaining states become almost stable ("trapped"). The short-lived state is seen as a broad resonance in a given channel and can be physically identified with a fast direct process. Thus, contrary to the naive expectation, the system avoids the overlap regime. The restructuring occurs in the narrow region and resembles a phase transition. It turns out that in the one-channel problem there is no place for strongly overlapped resonances and corresponding Ericson fluctuations of the cross sections. The phase transition with the width collectivization was first clearly seen in the calculations for the nuclear shell model in continuum [3]. Analogous phenomena exist in atomic, molecular, solid state and particle physics. An embryo signature of the trend to the width collectivization, level $attraction$ and width $repulsion$ due to the coupling via continuum, opposite to the usual mixing by a hermitian perturbation, is seen in a number of two-dimensional unstable systems, as a doublet of excited states in ^{8}Be, neutral kaons, and $\rho - \omega$ meson interference [4].

The coupling of intrinsic states through continuum introduces a new energy and time scale. The shortest time, $\tau_1 \simeq \hbar/\Gamma_D$, where Γ_D is the width of the broad ("Dicke") resonance, characterizes the fast direct reaction. In the strong coupling regime, Γ_D exceeds the energy range $a \sim ND$ covered by N intrinsic states strongly coupled to a given decay channel. The corresponding time $\tau_2 \simeq \hbar/a$ is the fragmentation time for the doorway state excited in a reaction as a superposition of quasistationary intrinsic states. The next scale $\tau_3 \simeq \hbar/D$ is the Weisskopf recurrence time for the wave packet constructed of typical intrinsic states. This time is necessary for resolving the individual fine structure levels. Finally, the small width Γ_t of trapped states determines their lifetime $\tau_4 \sim \hbar/\Gamma_t$. At strong coupling, $\kappa \gg 1$, the hierarchy of the time scales looks like

$$\tau_1 : \tau_2 : \tau_3 : \tau_4 = 1 : \kappa : N\kappa : N\kappa^2. \tag{3}$$

Since the trapped states live longer than the recurrence time, they certainly correspond to a fully equilibrated compound system.

As an example we consider [5] an exactly solvable model of a periodic chain of N potential wells $(n = 1, 2, \ldots, N)$ coupled by a (hermitian) interaction of a hopping or tunneling type. This can be thought of as a simple model of electron transport in a one-dimensional system. However the chain is open, and the escape amplitude for the electron from the site n is A_n. If the identical isolated wells support the level at energy ϵ, the effective non-hermitian hamiltonian can be written as a matrix in the site representation,

$$\mathcal{H}_{mn} = \epsilon\, \delta_{mn} + v(\delta_{m,n+1} + \delta_{m,n-1}) - \frac{i}{2} A_m A_n. \tag{4}$$

In a more general sense, this is the model for the migration of the excitation of a quantum system along the chain of states of progressive complexity. This is what is going on in a nucleus excited by an external field into a simple doorway state which is being mixed by an intrinsic interaction with a hierarchy of more complicated states. At each state of this chaotization process, the decay outside is possible.

With no continuum decay, the stationary states are delocalized Bloch waves $|q\rangle$ with the wave vector q (complicated shell model combinations of many exciton classes in a nuclear interpretation). Through the continuum, the overlapping intrinsic states are additionally coupled to each other. The states are in fact the resonances embedded in the continuum, and this openness of the system in turn influences

the internal dynamics. The problem can be exactly solved under different assumptions concerning the decay amplitudes A_n.

Let the decay probabilities be the same for all sites, $A_n = A = \text{const}$.

Fig. 1 shows the evolution of the complex energies for $N = 10$ wells. The Bloch wave decay amplitudes B_q have a maximum value for $q = 1$ and monotonously decrease with q. As the continuum coupling constant κ increases, the highest level in the band slightly reduces its real energy (level attraction) and, after a critical value of κ, rapidly absorbs the total width. Other states initially acquire small widths (perturbational region) and then return to the trapped regime.

In the case of the periodic chain open at the ends, so that only the outer wells have access to the continuum (analog of the situation when only the doorway state has a chance to decay into a specific channel while the higher complexity states lead to evaporation or breakup), the width distribution is shown in Fig. 2 for few values of κ. At low value of κ the result confirms the intuitive picture: the decay actually singles out the specific components of the quasistationary state which are best connected to the appropriate channels. This is precisely the argument used in the derivation of the Porter-Thomas distribution for the width distribution of the compound states.

The strong continuum coupling changes the intrinsic structure by introducing important correlations into the wave functions of quasistationary states. The short-lived Dicke states are located, contrary to the previous model, near the middle of the spectrum. In the coordinate (site or "shell model") representation, the Dicke states are almost pure edge states. The irreversible decay outside occurs faster than the hopping to the next well. The transition between the two regimes shows up as the segregation of the two short-lived surface-localized states from the band of delocalized Bloch waves (compound states).

This example can be considered as a prototype of realistic problems with excitation of intrinsic states by external fields. Not all modes are equally accessible for external agents. The doorway states serve as the first stages of the excitation process. The probability of excitation can be suppressed if the doorway states are short-lived so that they decay prior to the further transfer of the excitation. Two phenomena in nuclear reactions were recently studied from this viewpoint [6]. It is known that the Coulomb interaction violates isospin invariance of nuclear forces, and the effect is expected to be stronger at high level density. However, the isospin mixing occurs mostly in the nuclear interior. Heavy ion reactions first excite doorway states with a given isospin value which then can be mixed with the intrinsic states of different isospin. As excitation energy increases, the decay width of doorway states increases as well. As a result, they have no time for intrinsic mixing, and we observe the restoration of isospin purity predicted long ago and recently confirmed experimentally. A similar mechanism is, at least partly, responsible for the apparent disappearance of the collective strength of the giant dipole resonance in highly excited nuclei. The complicated doorway states excited in a heavy ion collision do not have time to excite, through the intrinsic mixing, the collective vibration. The entire scenario turns out to be quite general, being determined by such basic principles as hermiticity of the total hamiltonian (including the continuum states) and unitarity of the scattering matrix.

References

1. V.V. Sokolov and V.G. Zelevinsky, Phys. Lett. **B 202**, 10 (1988); Nucl. Phys. **A504**, 562 (1989); Ann. Phys. **216**, 323 (1992).
2. R.H. Dicke, Phys. Rev. **93**, 99 (1954).
3. P. Kleinwachter and I. Rotter, Phys. Rev. **C32**, 1742 (1985).

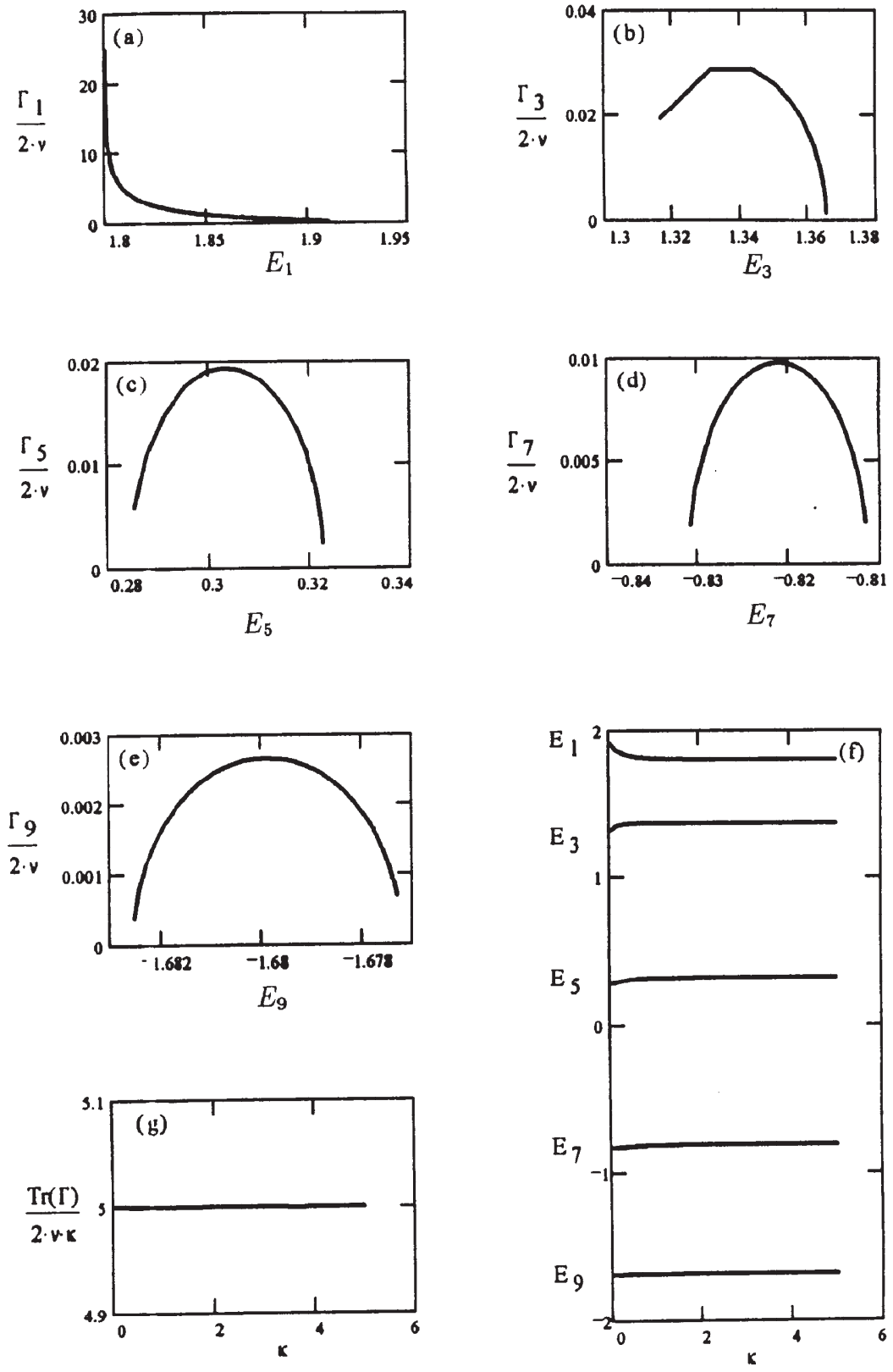

Figure 1: Trajectories of complex energies of odd levels in the case of $N = 10$ wells and uniform decay in the interval of κ between 0 and 5. The path of the highest energy state, panel a, begins at the lower right and proceeds to the upper left. The paths of lower states, panels *b*-*e*, start at the left and proceed to the right. The real energies are almost constant as a function of κ, panel *f*. The constant value of the summed width (the trace of the imaginary part of the hamiltonian) is used to check the calculations, panel *g*.

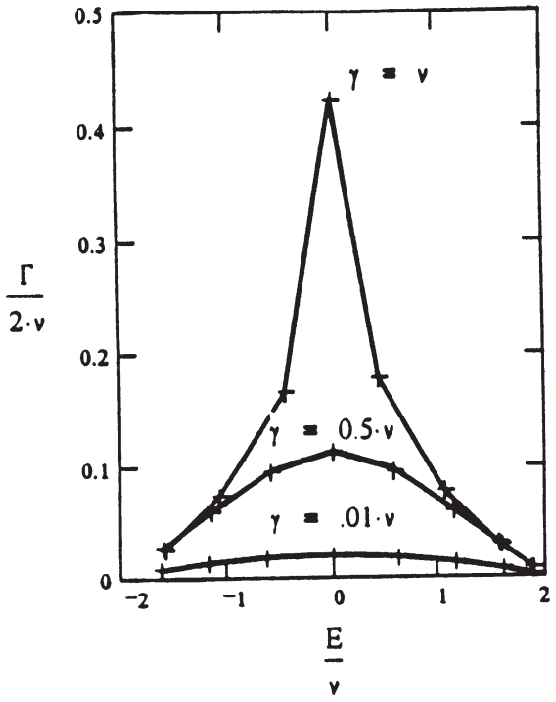

Figure 2: Formation of the Dicke resonance for the case of decay at the ends, $\gamma_L = \gamma_R = \gamma$. The state in the middle of the spectrum accumulates the increasing width as the decay probability increases from $\gamma = 0.01v$ to $\gamma = v$.

4. P. von Brentano, Phys. Rep. **264**, 57 (1996).
5. J. Armstrong and V. Zelevinsky, Preprint MSUCL-1098, April 1998.
6. V.V. Sokolov and V. Zelevinsky, Phys. Rev. **C56**, 311 (1997).