SOLUTION OF THE REDUCIBILITY PUZZLE

W. Bauer and S. Pratt

During the last decade, evidence has been mounting that nuclear matter undergoes a phase transition in the nuclear fragmentation process. From general consideration regarding the elementary nucleon-nucleon interaction (repulsive at short and attractive at intermediate distances), we expect the nuclear phase-diagram to show a Van-der-Waals "liquid-gas" phase transition of first order, terminating in a second-order transition at the critical point.

If one wants to gain a fundamental understanding of the fragmentation process that goes beyond simple equilibrium model descriptions of the phenomena, then a proper description of the origin and time evolution of fluctuations is essential, in particular if one wants to understand why particular molecular dynamics codes produce fragments (or not!), and what their connections to the fundamental processes of nuclear fragmentation are.

In this light, the recent findings of Moretto *et al.* are all the more surprising [1]. This group found that the probability P_n of emitting n intermediate mass fragments (IMFs) follows a binomial distribution

$$P_n(m,p) = \frac{m!}{n!\,(m-n)!} p^n (1-p)^{m-n} \tag{1}$$

The parameters m and p are related to the average and variance of the distribution.

$$\langle n \rangle = \sum_{n=0}^{\infty} n\, P_n(m,p) = m \cdot p \tag{2}$$

$$\sigma^2 = \sum_{n=0}^{\infty} (n - \langle n \rangle)^2\, P_n(m,p) = m\, p\, (1-p) \tag{3}$$

This result suggest that one may interpret the parameter p as the elementary probability for the emission of one fragment and the parameter m as the total number of tries. This would indicate that the problem of multi-fragment emission is reducible to that of multiple one-fragment emission. The claim for reducibility and its interpretation as the consequence of a simple barrier penetration phenomenon was further strengthened by the observation that $\ln(p^{-1})$ has a linear dependence on $1/\sqrt{E_t}$, where E_t is the total transverse energy, $E_t = \sum_l E_{kl} \sin^2\theta_l$. Finally, the same scaling was found for different beam energies and different projectile-target combinations.

Here we show that these observed patterns arise quite naturally from two main effects [2]:

- The finite size (charge) of the emitting system limits the fluctuations in the number distribution of intermediate-mass fragments stronger than it does the average. Thus, even if IMFs would be emitted with a Poissonian probability distribution in a infinite system, the probability distribution in a finite system will be sub-Poissonian and can be fit by a binomial distribution. How far away from the Poissonian limit the distribution is depends primarily on the size of the emitting system.

- There is a direct correlation between the number of detected charges in an event (= size of the emitting system) and the total transverse energy. This is true for practically all detection systems, even the ones that employ 4π solid angle coverage.

We begin our study by generating power-law distributed random fragmentation events. This is accomplished by determining the charge of individual fragments with a probability distribution proportional to $Z^{-\tau}$, where Z is the fragment charge, and τ is the power-law exponent. For definiteness, we wish to generate events with exactly $Z_{\rm sys}$ charges. If an event has less than $Z_{\rm sys}$ charges, we add another fragment; if it has more than $Z_{\rm sys}$ charges, we throw it out. For an infinite system, we would expect the multiplicity distributions for individual fragments of a given Z to follow a Poisson distribution. However, as mentioned above, for finite systems the probability distributions for individual IMFs and for all IMFs are better fit by binomial distributions.

What is the dependence of $Z_{\rm sys}$ on the transverse energy, E_t, in the experiments of Moretto *et al.*? The dominant feature is the linear rise of the mean value of $Z_{\rm sys}$ with E_t,

$$\langle Z_{\rm sys}(E_t)\rangle \approx 2 + 0.092\ E_t/{\rm MeV} \tag{4}$$

for values of E_t less than 0.7 GeV, and the saturation of $Z_{\rm sys}$ for larger values.

The width of the $Z_{\rm sys}(E_T)$-distribution is significant, on the order of 10 units of charge. If we wish to construct the probability distributions of intermediate mass fragments by using our knowledge of the dependence of the binomial parameters p and m on the system size, and the dependence of the system size on transverse energy, we have to integrate over the experimentally measured width of the $Z_{\rm sys}(E_T)$-distribution.

The outcome of this procedure is a (nearly) monotonic dependence of the extracted binomial parameter p on E_t. In fact, to very good approximation, a plot of $\ln p^{-1}$ vs. $1/\sqrt{E_t}$ shows an approximately linear rise. This, however, is *not* the consequence of some kind of thermal scaling. Instead, it is purely a consequence of the variation of the size of the emitting system as a function of the transverse energy, and with it a change in the effective parameter p in the binomial probability distribution.

We can obtain more-or-less complete agreement with the experimental data, if we allow the power-law parameter τ for the fragment mass distribution to vary with impact parameter and with beam energy. This variation is a well-documented experimental fact [3,4] the experimentally observed value of τ increases with impact parameter and therefore falls with transverse energy. If we assume

$$\tau(E_t) = 3.5 - E_t/(0.5\ {\rm GeV})\ , \tag{5}$$

then we get the result displayed in fig. 1.

Our calculations are represented by the plot symbols. The error bars are statistical and computed on the basis of 2×10^4 events for each point. The solid line is a fit to the experimental results of Moretto *et al.* As one can see, there is very good agreement. Assuming other functional dependences of τ of E_t may even yield better results. This agreement, however, is not quite as relevant as the main message we wish to impress on the reader: The universal scaling of $\ln p^{-1}$ vs. $1/\sqrt{E_t}$ is almost exclusively due to the finite size of the system emitting the fragments and the dependence of the measured value of the transverse energy on that size.

Even though the main message of the present note is that the E_t-dependence of the extracted binomial parameters of the fragment multiplicity distributions can be explained rather straightforwardly, we do not wish to convey the message that there is no interesting information that one can extract from this type of analysis. For instance, the effects of varying system size could be eliminated with utilization of completely reconstructed fragmentation events. For these types of events, percolation models predict a transition between sub- and super-Poissonian fluctuations near the percolation threshold. This type of behavior is not expected in a sequential model. Once the kind of correlations discussed by us above are removed, then this type of fluctuations analysis should yield insightful information about the character of the nuclear fragmentation phase transition.

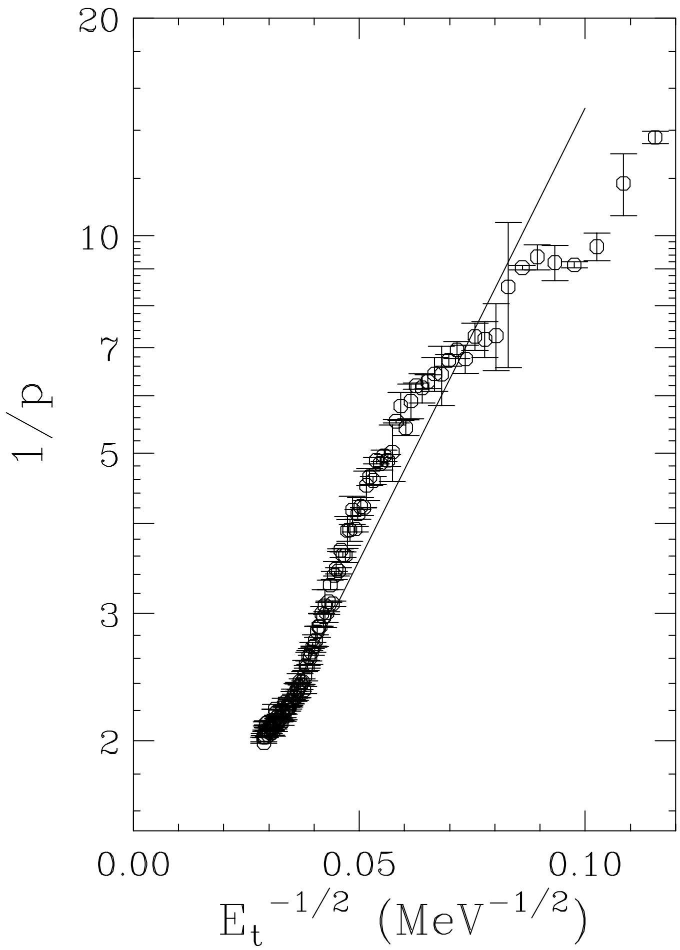

Figure 1: Dependence of the binomial parameter p of the IMF distribution on the transverse energy, assuming that the effective power τ of the fragment probability increases linearly with transverse energy. Plot symbols with error bars represent our calculations, the solid line is a fit to the experimental data of Moretto and collaborators.

References

1. L.G. Moretto *et al.*, Phys. Rev. Lett. **71**, 3935 (1993); L.G. Moretto *et al.*, Phys. Rev. Lett. **74**, 1530 (1995); L. Phair *et al.*, Phys. Rev. Lett. **77**, 822 (1996); L.G. Moretto *et al.*, Phys. Rep. **287**, 249 (1997).
2. W. Bauer and S. Pratt, submitted to Phys. Rev. C.
3. C. Williams *et al.*, Phys. Rev. C **55**, R2132 (1997).
4. T. Li *et al.* Phys. Rev. Lett. **70**, 1924 (1993); T. Li *et al.* Phys. Rev. C **49**, 1630 (1994).