MASS DEPENDENCE OF THE EFFECTIVE CHARGES IN THE $sd$-SHELL

M. Fauerbach$^{a}$, P.D. Cottle$^{a}$, K.W. Kemper$^{a}$, T. Glasmacher, R.W. Ibbotson, B. Pritychenko, H. Scheit$^{a}$, M. Steiner

In order to map out a possible mass and isospin dependence of the effective charges in the $sd$-shell, we performed an intermediate energy Coulomb excitation experiment with the semi-magic, proton-rich nucleus $^{38}\text{Ca}$. The transition between the first excited state ($2^{+}$, $E^{*} = 2206$ keV) and the ground state ($0^{+}_{gs}$) was studied, and the reduced transition strength and the proton transition matrix element were deduced. The nucleus $^{38}\text{Ca}$ was chosen for this study, as it is at the upper mass end of the $sd$-shell, and will together with already existing high precision data for the lower mass end of the shell most clearly show a systematic trend in the effective charges. Our study also helps to fill the void of available high quality data for proton-rich nuclei in this mass region. We compare our data to theoretical shell model predictions with mass dependent, and mass independent effective charges, and to experimental values of the neutron-rich mirror nucleus $^{38}\text{Ar}$.

Shell model calculations have been very successful in calculating transition matrix elements for a variety of nuclei in the $sd$-shell (see eg 1, 2, 3). Whereas the ‘total’, measured proton (neutron) transition matrix elements ($M_{p(n)}$) can contain contributions from excitation of all protons (neutrons) in the nucleus over indefinitely many shell-model orbits, this is --due to computational limitations-- not the case for the ‘model space’ transition matrix elements ($A_{p(n)}$). Due to this truncated model space, one has to renormalize the calculated transition matrix elements, in order to be able to reproduce the ‘total’, measured matrix elements. Therefore, the concept of effective charges was successfully introduced in order to reproduce core polarization effects generally associated with electric transitions. In this framework, the measured and calculated matrix elements are related via:

\[
M_{p} = (A_{p}e_{p} + A_{n}e_{n}) \quad (1)
\]
\[
M_{n} = (A_{n}e_{p} + A_{p}e_{n}) \quad (2)
\]

Where $e_{p,n}$ represents the effective proton (neutron) charge in units of $e$. In principle one would expect the effective charges to be state and mass dependent. However, Brown et al. 1 found no evidence for a state or mass dependence when comparing their shell-model calculations to experimental data in the $sd$-shell. However, the authors also point out the lack of available data from proton-rich nuclei, which makes a firm conclusion difficult. Alexander, Castel and Towner 2, on the other hand see evidence for slight ‘uniform’ increase in the ratio of the proton to neutron polarization charge when going from the lower to the higher mass end of the $sd$-shell.

A 80 MeV/nucleon $^{48}\text{Ca}$ beam from the K1200 cyclotron at the National Superconducting Cyclotron Laboratory irradiated a 202 mg/cm$^{2}$ target of $^{9}$Be located at the mid-acceptance target position of the A1200 fragment separator 4. The energy spread of the resulting $^{38}\text{Ca}$ fragments was limited to ±1% with an aperture and separation was obtained by placing a thin, achromatic wedge ("Al, 64 mg/cm$^{2}$") at the second dispersive image of the A1200. A ‘cocktail’ beam containing several fragment species was used to perform the experiment in order to study other nuclei in the vicinity simultaneously. This could be done, as the counting rate was not a limiting factor, and the fragment identification was unambiguous. About 20% of the mixed beam was $^{38}\text{Ca}$ ($\approx 12\text{ kfps}$). The average energy of the incoming $^{38}\text{Ca}$ particles was 56.1
MeV/nucleon. The time-of-flight was measured on an event-by-event basis over the approximately 14 m long flight path from a thin fast plastic scintillator located after the exit of the A1200 to the zero-degree detector. After passing through two x-y position sensitive parallel-plate avalanche counters (PPAC) 5, the beam impinged on the secondary target ($^{65}$Au, 184.1 mg/cm$^2$). The scattered ions were stopped in the zero-degree detector, which consisted of a fast-slow plastic phoswich detector. This detector defines a half-cone opening angle of $\theta_{lab} < 4.0^\circ$. The energy and time resolution in the zero-degree detector allowed an unambiguous isotopic identification of the fragments. The secondary target was surrounded by an array of 38 position sensitive NaI(Tl) detectors arranged in three concentric rings around the target and shielded from background photons by 16.5 cm thick walls of low-background lead. A more detailed description of the experimental setup and analysis procedures can be found in Refs. 6, 7, 8, 9, which also illustrate the Doppler-shift correction technique.

One of the beam contaminants was $^{32}$S, which has a ($^{2+} \rightarrow 0^+_\pi$) transition with an energy of $E_{\gamma} = 2230.5$ keV. This is very close to the ($^{2+} \rightarrow 0^+_\pi$) transition energy in $^{38}$Ca ($E_{\gamma} = 2206$ keV). As the reduced transition strength in $^{32}$S is known to high precision 10 we can use our measured $\gamma$-ray intensities combined with the number of incoming particles to deduce the transition strength in $^{38}$Ca without having to rely on efficiency measurements. The Doppler-corrected $\gamma$-ray energy spectra, recorded under the condition that a $^{38}$Ca ($^{32}$S) fragment was detected in the zero degree-detector, are shown in the top (bottom) part of Fig. 1. The photopeaks centered around $\gamma$-ray energies of 2206 keV and 2230 keV in the projectile frame ($^{38}$Ca ($^{32}$S)) -- corresponding to the $^{2+} \rightarrow 0^+_\pi$ transitions are clearly visible for both beams.

We measure a reduced transition strength $B(E2, 2^+_i \rightarrow 0^+_\pi) = (12.7 \pm 2.96) e^2f m^4$ for $^{38}$Ca. From this reduced transition strength we extract the proton transition matrix element $M_p$ via the relation:

$$M_p^2 = (2J_i + 1) B(E2, J_i \rightarrow J_f) \frac{1}{\epsilon}$$

(3)

Where $J_{i(f)}$ stands for the initial (final) spin. Our result of $M_p = (7.93 \pm 0.97) f m^2$ has to be compared
to the results of shell-model calculations with mass independent and mass dependent effective charges.

Brown et al. 1 use mass independent effective charges \( e_p = 1.15e, e_n = 0.45e \) and a finite-well, ‘local Woods-Saxon’ potential (see 1 for details), and derive a proton transition matrix element of \( M_p = 3.1 \text{ fm}^2 \), which is more than a factor of two smaller than the measured value. A possible explanation for this discrepancy might be the use of the (simpler) finite well potential for this calculation. This seems to effect the nuclei at the upper end of the \( sd \)-shell more than the lighter \( sd \)-shell nuclei, which can be seen clearly for the mirror nucleus \(^{38}\text{Ar}\). For \(^{38}\text{Ar}\), Brown et al. performed both a ‘full blown’ calculation, using a harmonic oscillator potential, and one with a ‘local Woods-Saxon’ potential (quoted above). Whereas, they get excellent agreement with the experimental value for \( M_p \) when using the ‘full’ calculation, the value calculated using the ‘local Woods-Saxon’ potential, underestimates the experimental \( M_p \) by 25%. For \(^{40}\text{Ca}\) this effect might even be enhanced due to the relatively small proton binding energy in this nucleus.

Alexander, Castel, and Towner 2 analyzed a similar set of data, however, they derived not only mass dependent effective charges, but also found evidence for a isospin \((T_z)\) dependence of the effective charges. The different derived effective charges and the corresponding transition matrix elements can be found in Table I. A comparison to our experimental value shows very good agreement with the theoretical calculations using the isospin dependent effective charges.

In conclusion, our measurement of the reduced transition strength in the proton-rich nucleus \(^{40}\text{Ca}\) provides essential information about the mass, and isospin dependence of the effective charges in the \( sd \)-shell. Our deduced proton transition matrix element for \(^{40}\text{Ca}\), is more than a factor of two larger than the value deduced by Brown it et al. 1, using mass, and isospin independent, constant effective charges. We find good agreement with the theoretical values derived by Alexander et al. 2, when using their isospin dependent description for the effective charges. Furthermore, our data indicates that a combination of the isospin dependence with a mass dependence of the effective charges would lead to an even better agreement, between the theoretical and experimental data. Finally, our result also seems to confirm the previously suggested decrease of the isovector effective charge with mass. The lack of high quality data of proton-rich nuclei in this mass region made a conclusion about the mass dependence of the isovector effective charge prior to this measurement impossible. We were able to contribute substantially in filling this void, however, further investigation of other proton-rich nuclei would be helpful in order to adequately describe this mass dependence. A more detailed discussion about this subject can be found in 11.

**Table I**: Extracted effective charges from Ref. 2, and the corresponding transition matrix elements. The first two lines show the values extracted from fits to nuclei with positive (negative) \( T_z \). The third line shows the values without a \( T_z \) separation, and the fourth line shows the results for a mass dependent fit. Here, \( n \) denotes the number of nucleons outside the \(^{16}\text{O} \) core \((n = 22 \text{ in our case})\).

<table>
<thead>
<tr>
<th></th>
<th>( e_n ) [e]</th>
<th>( 1 + e_p ) [e]</th>
<th>( M_p ) (theory) [fm(^2)]</th>
<th>( M_p ) (exp.) [fm(^2)]</th>
<th>( M_p ) (theory) [fm(^2)]</th>
<th>( M_p ) (exp.) [fm(^2)]</th>
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<tbody>
<tr>
<td>( T_z &gt; 0 )</td>
<td>0.403</td>
<td>1.169</td>
<td>3.3</td>
<td>7.97(93)</td>
<td>10.2</td>
<td>10.9(4)</td>
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<tr>
<td>( T_z &lt; 0 )</td>
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<td>0.945</td>
<td>6.2</td>
<td>8.2</td>
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<tr>
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<td>1.030</td>
<td>4.6</td>
<td>9.0</td>
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<tr>
<td>A dep.</td>
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<td>1.188 + 0.0035n</td>
<td>4.8</td>
<td>11.0</td>
<td></td>
<td></td>
</tr>
</tbody>
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a. Department of Physics, Florida State University, Tallahassee, FL 32306, USA

References
7. T. Glasmacher, P. Thirolf, and H. Scheit, to be published.
11. M. Fauerbach et al., to be published.