Studies of the thermodynamic properties of nuclear matter require accurate methods of temperature determination. Double-ratios of isotope yields which cancel out chemical potential effects offer a particularly promising technique of temperature determination, known as the Albergo-method [1].

The open and solid circular points in Fig. 1 show the measured isotopic yield ratios, d/t (left panel) and $^3$He/$^4$He (middle panel), and the double ratio, RH-He (right panel), for central $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn collisions, respectively. For both reactions, $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn, the measured double ratio is the same, $R_{H-He} = 10$, within experimental errors. This observation is consistent with the results of ref. [2]. Thus, the isotope double ratio is independent of the isospin of the system as is required for consistency with equilibrium that is necessary for fragmentation models. This value for the RH-He double ratio is obtained from the two isotope ratios, d/t and $^3$He/$^4$He, both of which depend, as expected [3], strongly on the proton-to-neutron ratio of the reaction involved. Both the d/t and the $^3$He/$^4$He isotope ratios are about 40% greater for the $^{112}$Sn + $^{112}$Sn reaction than for $^{124}$Sn + $^{124}$Sn. As a consequence, the RH-He double ratio has the same value for both systems.

This confirmation of the insensitivity of the RH-He double ratio to the N/Z ratio of the emitting source is, however, insufficient to ensure the existence of a well defined freeze-out temperature. To illustrate this point, we performed calculations with the expanding emitting source (EES) model of ref. [4]. In these calculations, we assumed no initial collective expansion velocity, an initial density equal to that of normal nuclei, an initial excitation energy per nucleon of $E^*/A = 8$ MeV, and an initial temperature of $T = 11.16$ MeV. This particular value of $E^*/A$ corresponds to roughly 80% of the available energy in the center of mass system. The default parameters of the EES models were used: $\varepsilon_F = 30$ MeV and $K = 144$ MeV, where $\varepsilon_F$ is the Fermi energy and K is the restoring force. The sensitivity to the unknown mass of the sources was assessed by investigating two limiting cases, corresponding to initial sources containing all nucleons from target and projectile (denoted as $^{224}$Fm and $^{248}$Fm) or, alternatively, only one-half the total number of protons and neutrons (denoted as $^{112}$Sn and $^{124}$Sn).

Predictions of the EES model are shown by rectangular symbols in Fig. 1, where calculated values are shown for both the individual d/t and the $^3$He/$^4$He ratios and also for the double ratio RH-He. Solid (open) symbols indicate calculations for the initial neutron-to-proton ratio of N/Z = 1.48 (1.24). The vertical size of the rectangular symbols indicates the variation of the model prediction for the two extreme source sizes. (Since the d/t ratio is nearly independent of source size, the rectangular symbols shrink to lines, with the N/Z = 1.48 case having the lower values.) The model provides predictions for both primary yields and additional contributions from the decay of low-lying resonances of He, Li, and Be isotopes. The model predictions for the respective ratios from both the primary (p) and final (f) yields are indicated by the same symbols and compared with the experimental values indicated by circles. The model predicts significant feeding contributions to all isotopes of light nuclei. For the case of $^4$He, the predicted final yield is dominated by contributions from sequential decay.

Most remarkably, the EES model predicts that the RH-He double-ratio is very insensitive to the N/Z ratio of the emitting source and only slightly sensitive to its size (see right-hand column of Fig. 1). This insensitivity to the N/Z ratio is predicted for the double ratios of both primary and final particle yields. Consistent with previous findings, however, the magnitude of the calculated double ratio is strongly altered by sequential feeding from particle unbound decays [5]. Since the yields of particle unbound primary fragments and their subsequent decay have not been measured in this experiment, the excellent
agreement between measured and predicted final $R_{H\text{-He}}$ double-ratios must be viewed with caution and could be fortuitous.

Fig. 1: Single yield-ratios (left panel), $d/t$ and $^3\text{He}/^4\text{He}$ (times 10), and double-ratio $R_{H\text{-He}}$ (right panel) for central $^{112}\text{Sn} + ^{112}\text{Sn}$ (open symbols) and $^{124}\text{Sn} + ^{124}\text{Sn}$ (solid symbols) collisions at $E/A=40$ MeV. Experimental data (d) are shown as circles; statistical errors are smaller than the size of the data points. Rectangular symbols represent predictions of the EES model for sources with $E^*/A = 8$ MeV. The vertical size of the symbols depicts the difference in prediction for a source containing all (upper edge) or only one-half (lower edge) of the protons and neutrons contained in the combined projectile and target system. Theoretical predictions for primary and final particle ratios are labeled p and f.

Fig. 2 shows the predicted time-dependence of light particle emission rates and instantaneous temperatures for the sources $^{224}\text{Fm}$ (dashed curves) and $^{248}\text{Fm}$ (solid curves) initially at $E^*/A = 8$ MeV. The figure only shows the time dependence for $t < 200$ fm/c when most of the interesting activity occurs and more than 80% of the particles are emitted. The final yields, shown in Fig. 1, were calculated with the default values of the model (over 800 fm/c). As expected, both sources are predicted to exhibit rather similar (though not identical) temperature curves with a local maximum at $t \approx 150$ fm/c caused by a maximum in the density oscillation of the source. With the exception of $^4\text{He}$, the predicted light-particle emission rates exhibit a similar time dependence including a maximum at $t \approx 150$ fm/c, with the absolute rates depending on the N/Z ratio of the initial source. In contrast, the $^4\text{He}$ rates exhibit a distinctly different time dependence with a pronounced maximum at $t \approx 60$ fm/c (plus some less interesting damped oscillations at later times). The EES model predicts an enhanced emission of strongly bound $^4\text{He}$ nuclei and IMFs (at $t \approx 60$ fm/c) relative to nucleons and the other light particles when the source has expanded to a low density minimum. As a consequence, light particles (including $^3\text{He}$) sample the temporal evolution of the expanding source with different time-dependent weights than do IMFs and $^4\text{He}$ nuclei. Within the EES model, double ratios involving (time-integrated) yields of $^3\text{He}$ and $^4\text{He}$ nuclei therefore provide an “apparent temperature”, but this value cannot be related to the
temperature at some average freeze-out time because $^3$He and $^4$He nuclei are preferentially emitted at different stages of the reaction [5].

![Graph showing temporal evolution of p, d, t, $^3$He, and $^4$He emission rates and source temperatures predicted by the EES model during the early stage of the reaction where most of the emission takes place. Dashed and solid curves depict calculations for initial sources $^{224}$Fm and $^{248}$Fm, respectively, with $E^*/A = 8$ MeV.]

In summary, we have investigated the emission of light particles in central collisions of $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn at $E/A = 40$ MeV and confirmed the insensitivity of the $R_{^3\text{H}-^4\text{He}}$ double-ratio to the neutron-to-proton ratio of the emitting source predicted by grand canonical ensembles. This insensitivity is, however, also predicted by the EES model which incorporates time-dependent cooling by expansion and evaporation, plus feeding from particle unbound states of primary emitted fragments. Thus, the observation of a double-isotope ratio independent of the $N/Z$ ratio of the reacting system, while necessary, is not sufficient to ensure the existence of a well-defined freeze-out temperature.

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