First evidence for strong deformation at or near the $N = 20$ shell closure was obtained from mass measurements of $^{26-35}\text{Na}$ [1]. More recently, intermediate energy Coulomb excitation measurements of the $B(E2; 0^+ \rightarrow 2^+)$ value in the neutron-rich $N = 20$ nucleus $^{32}\text{Mg}$ yielded a result that implied a large degree of collectivity [2,3]. Shell model calculations including “intruder” configurations [4-6] as well as Hartree-Fock calculations [7] have been performed to gain an understanding of these observations.

In this report, we present the results of an intermediate energy heavy-ion scattering measurement of the $N = 20$ nucleus $^{31}\text{Na}$, which was performed at the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University. The $^{31}\text{Na}$ beam was produced via fragmentation of a primary 80 MeV/nucleon $^{48}\text{Ca}$ beam with intensity of 8 particle nA from the NSCL K1200 superconducting cyclotron. Fragmentation of the primary beam took place in a 376 mg/cm$^2$ thick $^9\text{Be}$ target located at the mid-acceptance target position of the A1200 fragment separator [8]. The energy of the $^{31}\text{Na}$ particles produced in the fragmentation reaction was 58.9 MeV/nucleon. A 702 mg/cm$^2$ thick gold foil was used as the secondary target. After passing through the gold foil, the secondary beam particles (including $^{31}\text{Na}$) were stopped in a cylindrical fast-slow phoswich detector which allowed nuclear charge identification of the secondary beam particles. While the secondary beam was run in a “cocktail” so that $^{31}\text{Na}$ was only a small fraction of the beam, time-of-flight measurements in the beam line and charge identification in the phoswich detector provided positive isotope identification. In total, $1.28 \times 10^{13}$ $^{31}\text{Na}$ ions were detected in the phoswich detector.

The NSCL NaI(Tl) array [9] was used to detect photons in coincidence with the scattered beam particles. The $\gamma$-ray spectrum of is shown in Figure 1. The top panel of Figure 1 shows the spectrum without correcting for the Doppler shift of $\gamma$-rays emitted from the $^{31}\text{Na}$ projectiles, while the lower panel includes this correction. The Doppler-shifted spectrum contains a $\gamma$-ray peak at 350$^\pm$20 keV. The ground state spin of $^{31}\text{Na}$ was determined by Huber et al. [10] to be $J = 3/2$, so we propose that the 350 keV $\gamma$-ray de-excites the first excited state (with $J = 5/2$) in a $K = 3/2$ rotational band built on the ground state. The yield in the 350 keV peak is analyzed to obtain a cross section of 115$^\pm$32 mb for producing this $\gamma$-ray. This analysis - which is described in detail in [11] - assumes that the angular distribution of this de-excitation $\gamma$-ray is that of a pure $M1$ transition (while the excitation is purely $E2$ in character). The present analysis also includes corrections for $\gamma$-ray absorption in the target, which in turn depends on the lifetime of the $J = 5/2$ state (and, therefore, on the range of locations of the $^{31}\text{Na}$ nuclei when they decay). The extreme cases of instantaneous decay and a lifetime so long that the Doppler shift correction no longer would work give efficiencies which are 11% lower and 5% higher, respectively.

It is quite likely that the $J = 7/2$ member of the rotational band is also strongly populated in the present reaction since the $E2$ matrix element connecting this state to the ground state would be comparable to that connecting the $J = 5/2$ state to the ground state. Therefore, the feeding of the 5/2 state by the 7/2 state must be considered when extracting a cross section for direct population of the 5/2 state from the yield of the 350 keV $\gamma$-ray. We estimate the energy of the $J = 7/2$ state using a shell model calculation which was performed for $^{31}\text{Na}$ using the same $sd - pf$ Hamiltonian and model space that was used in [12-14] for the neutron-rich Si, S and Ar isotopes. This calculation yields an energy of 1525 keV.
In-beam photon spectrum gated on $^{31}$Na. The top panel shows the spectrum without Doppler correction as measured in the laboratory with the $7/2^{-} \rightarrow 3/2^{+}$ transition in the gold target visible as a peak. The bottom panel shows the spectrum after event-by-event Doppler correction in the projectile frame.

Figure 1: In-beam photon spectrum gated on $^{31}$Na. The top panel shows the spectrum without Doppler correction as measured in the laboratory with the $7/2^{-} \rightarrow 3/2^{+}$ transition in the gold target visible as a peak. The bottom panel shows the spectrum after event-by-event Doppler correction in the projectile frame.

for the $J = 7/2$ state, 197 keV for the $J = 5/2$ state, and positive parity for the band. The results of our shell model calculations are in agreement with those of Caurier et al. [6], who predicted that transition energy for the first excited state in $^{31}$Na is $\sim$200 keV. The recent change [15] of a cross-shell interaction by the authors of Ref. [6] produced the $5/2^{+}$ state at 284 keV and the $7/2^{+}$ state at 1050 keV. While there is no evidence in the experimental spectrum for a $\gamma$-ray in the vicinity of the energy we expect for the $7/2 \rightarrow 5/2$ transition ($\sim$1175 keV or $\sim$700 keV), the experimentally observed background is consistent with the expected yields (based on considerations discussed below) of 4 (or 5) counts, respectively.

The coupled channels code ECIS88 [16] with an optical model parameter set determined for the $^{17}$O+$^{208}$Pb reaction at 84 MeV/nucleon [17] was used to calculate the nuclear contribution to the total cross section. We integrated the angular distribution out to the maximum scattering angle encountered in the experiment ($\theta_{\text{cm}} \approx 3.25^\circ$) and adopted a form factor corresponding to a static axial quadrupole deformation. This calculation includes two deformation parameters. In the rotational model [18], deformation parameters can be expressed as a function of $B(E2)$ values and rigid sphere radii as the following:

$$|\beta_2| = \frac{4\pi}{3} \sqrt{\frac{B(E2)}{Ze R_0^2}} \left( \frac{1}{J_J K_{20} |J_{I_J}|} \right),$$

where the corresponding Clebsch-Gordan coefficients for $3/2^+ \rightarrow 5/2^+$ and $3/2^+ \rightarrow 7/2^+$ transitions are $\sqrt{18/35}$ and $\sqrt{2/7}$, respectively. The radius $R_0$ is given by $R_0 = r_0 A^{1/3}$, where we take $r_0 = 1.20$ fm. The first, the "Coulomb deformation" $\beta_C$, reflects the deformation of the proton density in the nucleus and corresponds to the electromagnetic matrix element $B(E2; J_g \rightarrow I_f)$. For the $5/2^+$ state, the shell
model result \( B(E2;3/2 \rightarrow 5/2) = 196 \, \text{e fm}^4 \) gives a prediction of \( \beta_C = 0.51 \). In the case of the \( 7/2 \) state, the shell model calculation gives \( B(E2;3/2 \rightarrow 7/2) = 87.5 \, \text{e fm}^4 \), so that \( \beta_C = 0.46 \). The second deformation parameter in the calculation is the "nuclear matter deformation parameter" \( \beta_A \). The present shell model calculations yield results for neutron and proton transition multipole matrix elements which are different from the standard collective model picture. To account for these calculated matrix elements and the \( \text{rms} \) proton and neutron radii calculated in Ref. [19], \( \beta_A \) is set to 0.47 for both the \( 3/2 \rightarrow 5/2 \) and \( 3/2 \rightarrow 7/2 \) excitations. Motobayashi et al. [2] analyzed the \( ^{31}\text{Mg} \) data by using the standard rotational model (where \( \beta_C = \beta_A \)) to directly extract a quadrupole deformation parameter \( \beta_2 \). Such analysis produces \( \beta_2 = 0.59 \pm (8) \) for \( ^{31}\text{Na} \). The present shell-model calculation of \( ^{31}\text{Na} \), which has the higher neutron to proton ratio than \( ^{31}\text{Mg} \), predicts different deformations for protons and neutrons. This introduces an additional theoretical uncertainty which is reflected in the theoretical error. Thus the deformation parameter is \( \beta_2 = 0.59 \pm (8) \text{(experimental)} \pm (6) \text{(theoretical)} \). This deformation is comparable to that in the even-even \( N = 20 \) isotope \( ^{32}\text{Mg} \).

The ECI88 calculations using these deformation parameters yield cross sections of 54 mb for the \( 5/2 \) state and 27 mb for the \( 7/2 \) state. If 95\% of the decays of the \( 7/2 \) state go to the \( 5/2 \) state, the cross section for producing the 350 keV \( \gamma \)-ray would be 80 mb. Consequently, the shell model calculation is in agreement with the experimental result of 115 \pm 32 mb.

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