MICROSCOPIC DESCRIPTION OF ANHARMONICITIES IN SOFT COLLECTIVE NUCLEAR MODES

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In large amplitude nuclear collective motion the usual methods such as time-dependent mean-field or consideration of collective motion as a diffusion-like sequence of hoppings between different configurations [1] are not suitable for describing the individual stationary states. However, this spectroscopic problem emerges if a finite system has a soft collective mode. As the corresponding frequency ω goes to zero, a macroscopic system becomes unstable approaching the second order phase transition. In the vicinity of the transition the mean field manifests large scale fluctuations which dominate critical dynamics. In finite systems as nuclei or atomic clusters, phase transitions are smeared away. However, the very existence of the soft critical mode influences all observable properties. Typical example of such situation are low-lying quadrupole vibrations in transitional nuclei, that in such regions have neither static deformation nor well developed rotational band structure. Serious violations in the harmonic scheme along with microscopic arguments of rather general type [2] show that instead of the second order phase transition one should expect strong quartic anharmonic distortion of the potential as a function of the quadrupole coordinate. At the instability point of the Random Phase Approximation (RPA) which would indicate the phase transition in an infinite system the global stability is restored by quartic anharmonicity. The possible presence of a rather small maximum at zero deformation is not expected to create a deformation but is a cause of γ-instability [3]. Phenomenological models based on the picture of strong quartic anharmonicity turned out to be quite successful in reproducing regularities of experimental energies and transition probabilities for dozens of soft nuclei [4, 5, 6].

The goal of this work is to find the regular way for constructing the correct collective hamiltonian for the soft mode dynamics and calculating its parameters from the microscopic theory. Such a hamiltonian does exist if the low-lying states are known to be associated with the soft mode, as is presumably the case in transitional nuclei. The hamiltonian sought for is effective in the sense that all other degrees of freedom are projected onto the Hilbert space generated by the soft mode. Such an approach is used for macroscopic second order phase transitions. The difference is that here we need to get a complete quantum hamiltonian including the collective kinetic energy rather than free energy as a function of the order parameter only. On the other hand, our theory for a finite system can avoid scaling and renormalization problems.

Our projection formalism is based on identifying the form of the generalized density matrix (GDM) $R_{1\,2} = a_2^\dagger a_1$ in terms of collective variables (coordinate α and momentum π) and simultaneously mapping the dynamical evolution arising from the collective hamiltonian

$$H = \frac{\Lambda^{(0\,2)}\pi^2}{2} + \frac{\Lambda^{(2\,0)}}{2}\alpha^2 + \frac{\Lambda^{(3\,0)}}{3}\alpha^3 + \frac{\Lambda^{(1\,2)}}{4}[\alpha, \pi^2]_+ + \tag{1}$$

$$\frac{\Lambda^{(4\,0)}}{4}\alpha^4 + \frac{\Lambda^{(0\,4)}}{4}\pi^4 + \frac{\Lambda^{(2\,2)}}{8}[\alpha^2, \pi^2]_+ + \dots$$

to the microscopic equations of motion. This dynamical mapping along with kinematical properties of the GDM allows for the determination of the parameters Λ of the collective hamiltonian (1).

In the lowest order of mapping we obtain the Hartree-Fock equation relating the static mean field to the distribution of particles, while next (linear) order in collective variables reproduces the RPA. The

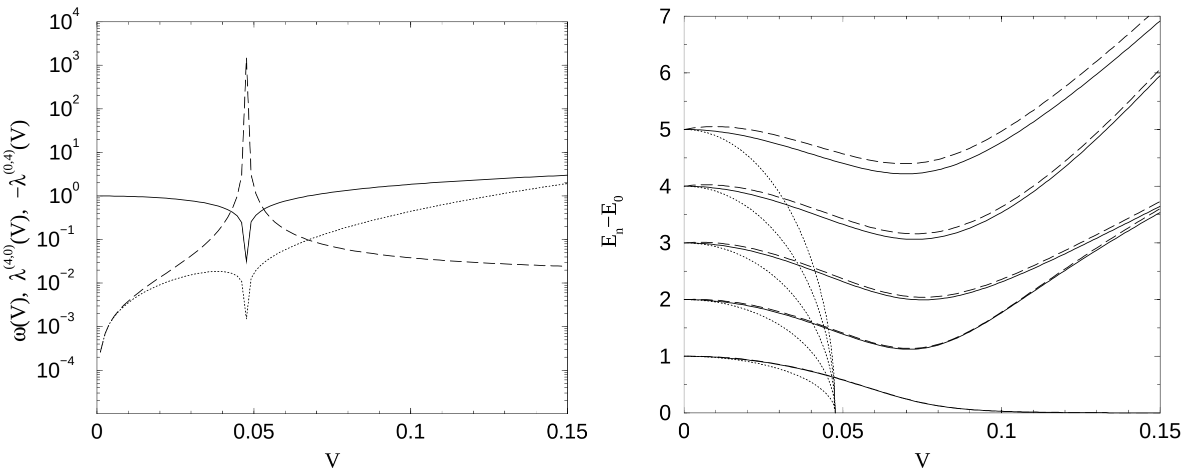

Figure 1:
Left: Plot shows the relative importance of collective frequency ω and non–zero anharmonic terms $\lambda^{(4,0)} = \Lambda^{(4,0)}/4\omega^3$ and $\lambda^{(0,4)} = \Lambda^{(0,4)}\omega/4$, in the LMG model with 20 particles. The logarithmic ordinate axis is plotted in the energy scale, that can also be compared to the distance between levels $\epsilon = 1$.
Right: The spectrum of first five excited collective states relative to the ground state in the LMG model with 20 particles. Solid line shows the exact solution, dotted line is RPA and dashed line anharmonic oscillator solution. In the latter the anharmonicity part with π^4 was ignored.

complication that one encounters is that each term experiences a renormalizing correction from the higher orders. For example, the renormalizing field fluctuations at the RPA level effect the particle distribution and Hartree-Fock mean field at the zeroth level of approximation. The self-consistent solution of all equations is quite complex even at low orders.

As a first test we have analyzed anharmonicities and their effects in the two-level Lipkin-Meshkov-Glick (LMG) model [7]. The special feature of this problem is that it has an intrinsic kinematically defined collective mode and a lot of symmetries in the Hamiltonian that reduce considerably complications associated with the exact solution. Fig. 1, left panel, shows the determined anharmonic terms in this model as a function of the interaction strength. It is clearly seen that when the RPA frequency goes to zero the quartic anharmonicity $\Lambda^{(4\,0)}$ becomes extremely important.

In the right panel of Fig. 1 we present a comparison of the exact LMG model spectrum (solid lines), RPA solution (dotted lines), and an improved anharmonic oscillator solution with the ignored divergent π^4 part. Introduction of an anharmonic term in this example produces a dramatic improvement as compared to RPA; the agreement of exact and approximate solutions is quite remarkable.

The further study of anharmonicities of quadrupole modes in separable models and in realistic systems is in progress.

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References

1. B.W.Bush, G.F.Bertsch, and B.A.Brown, Phys. Rev. C **45** (1992) 1709.
2. V.G.Zelevinsky, Int. J. Mod. Phys. **E2** (1993) 273.

3. M.Jean and L.Wilets, Phys. Rev. **102** (1956) 788.
4. O.K.Vorov and V.G.Zelevinsky, Yad. Fiz. **37** (1983) 1392 [Sov. J. Nucl. Phys **37** (1983) 830].
5. O.K.Vorov and V.G.Zelevinsky, Nucl. Phys. **A439** (1985) 207.
6. V.G.Zelevinsky, Soryushiron Kenkyu (Kyoto) **83** (1991) D176.
7. H. Lipkin, N. Meshkov, and A. Glick, Nucl. Phys. **62** (1965) 188.