NUCLEAR PAIRING PROBLEM WITH EXACT PARTICLE NUMBER CONSERVATION.

Alexander Volya, Vladimir Zelevinsky and Alex Brown

Pairing correlations play an important role in nuclear structure determining an essential contribution to binding energy, odd-even effects, single-particle occupancies, quasiparticle excitation spectrum, radiation and beta-decay probabilities, transfer reaction amplitudes, low-lying collective modes and moments of inertia [1, 2, 3]. The revival of interest to pairing correlations is related to studies of nuclei far from stability and predictions of exotic pairing modes [4]. Metal clusters, organic molecules and Fullerenes give another example of finite Fermi systems with possibilities of pairing correlations of superconducting type [5].

The conventional description of pairing in small systems usually employs the classical BCS [6] approach used in theory of superconductivity. This approximate solution has a very good accuracy for large systems and becomes exact in the asymptotic limit [7]. The major drawback of the BCS is the violation of particle number conservation, which gives rise to deviations from the exact solutions for small systems. Various ideas were suggested to correct this deficiency, such as the direct particle projection technique [8], number projection mean-field methods [9, 10], statistical description [11], and taking into account the residual parts of the Hamiltonian in the random phase approximation [12]. These methods have found only a limited number of practical applications; for some approaches the obtained results did not manifest the desired accuracy whereas for other methods the complications turn out to be almost on the same scale as for the exact solution by diagonalization. The Richardson method, described in the series of papers [13], provides a formally exact way of solving the pairing Hamiltonian with a constant effective pairing force. This method reduces the large-scale diagonalization of a many-body Hamiltonian in truncated Hilbert space to a set of coupled equations of a dimension equal to the number of valence particles. Recently, exact solutions have been approached by introducing sophisticated mathematical tools such as infinite-dimensional algebras [14]. Nevertheless the numerical complications currently limit the scope of applicability and the need for a good approximate theory still persists, especially because it can provide us with a convenient basis for calculating the effects of other parts of the residual interaction.

The goal of our work is to study a nuclear pairing problem with a particle conserving variational approximate solution that is formulated in the form of a recurrence relation in the number of particles N. For each step it is required to solve equations for only two variables, energy gap and chemical potential, as a function of the exact particle number, and thus even for large N the numerical procedure is quite fast. By making additional approximations this solution can be reduced to the BCS. The idea of the method goes back to the old papers [15, 16] and to the work [17] where the set of exact operator equations of motion was formulated by introducing the gauge angle conjugate to the particle number as a collective variable. In discrete space the corresponding equations are of recurrent type; the method was applied [17] to the so-called pairing rotations (a systematic change of pair separation energy). We construct an algorithm for the solution of recurrence relations derived from operator equations of motion with exact particle number conservation at each stage. In the well known degenerate model the solution coincides with the exact one. In a model with equidistant single-particle levels, our particle-conserving solution is compared to the exact solution and BCS solution, and an improvement over the BCS is observed. Finally, tin isotopes are considered as a realistic example, see figure 1. The exact calculations are based on the Nijm-I G-matrix

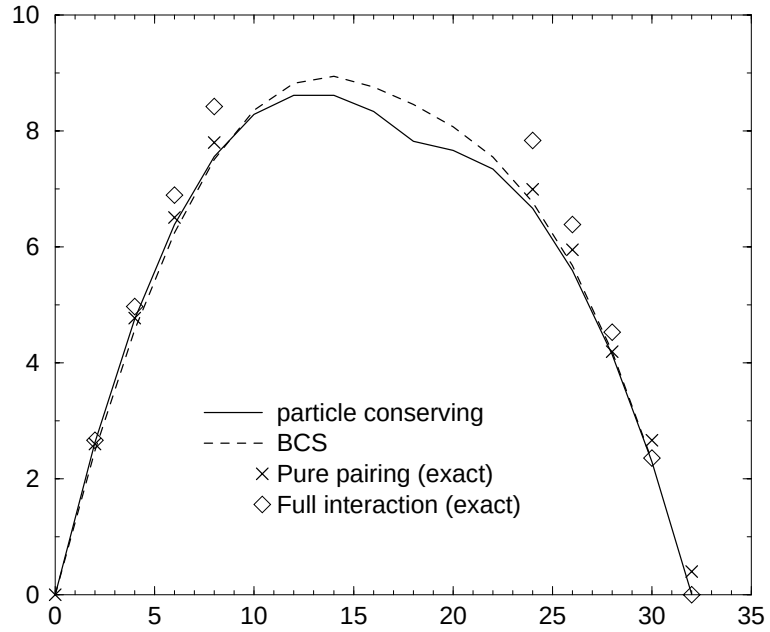

Figure 1: Correlation energy of tin isotops as a function of a number of holes in the shell. Particle conserving, BCS and exact solution from OXBASH are compared. Diamonds show the exact calculations with interactions in all momentum channels being included.

[18]. In our future work we intend to further investigate the particle conserving algorithm, study the weak coupling limit near the BCS breakdown and consider odd-particle systems with possible inclusion of higher seniority states in the formalism. Development of computer codes that would allow us to quickly account for pairing in all nuclei presents an important technical task. Questions of current interest such as isovector pairing and proton-neutron pairing effects can be viewed as possible promising avenues in the same direction.

References

1. S.T. Belyaev, Mat. Fys. Medd. Dan. Vid. Selsk. **31**, No. 11 (1959); in *Selected Topics in Nuclear Theory*, ed. F. Janouch (IAEA, Vienna, 1963) p. 291.
2. A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1974).
3. M. Hasegawa, S. Tazaki, Phys. Rev. C **47**, 188 (1993).
4. A.L. Goodman, Adv. Nucl. Phys. **11**, 263 (1979).
5. M. Barranco, S. Hernandez and R.M. Lombard, Z. Phys. **D 22**, 659 (1992).
6. J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
7. N.N. Bogoliubov, JETP **34**, 58 (1958); Physica **26**, 1 (1960).
8. A. K. Kerman and R. D. Lawson, Phys. Rev. **124**, 162 (1961).
9. H. J. Lipkin, Ann. Phys. **31**, 528 (1960).
10. Y. Nogami, Phys. Rev. **134B**, 313 (1964); Y. Nogami and I. J. Zucker, Nucl Phys. **60**, 203 (1964).
11. S. Das Gupta and N. De Takacsy, Nucl. Phys. **A240**, 293 (1975).
12. J. Bang and J. Krumlinde, Nucl. Phys. **A141**, 18 (1970).
13. R. W. Richardson, Phys. Lett. **3**, 277 (1963); **5**, 82 (1963); **14**, 325 (1965).

14. Feng Pan, J. P. Draayer and W. E. Ormand, Phys. Lett. B **422**, 1 (1998).
15. G.D. Dang and A. Klein, Phys. Rev. **143**, 735 (1966); Phys. Rev. **147**, 689 (1966).
16. G.D. Dang, R.M. Dreizler, A. Klein and C-S Wu, Phys. Rev. **172**, 1022 (1968).
17. S.T. Belyaev and V.G. Zelevinsky, Yad. Fiz. **16**, 1195 (1972).
18. G.N. White *et al.* Nucl. Phys. A **644**, 277 (1998).