

# SPACE CHARGE BEAM DYNAMICS STUDIES OF THE INJECTION LINE AND THE MAIN RING FOR THE UNIVERSITY OF MARYLAND ELECTRON RING PROJECT

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## 1 Introduction

The University of Maryland Electron Ring (E-Ring) [1] will operate in the space charge dominated regime. With electron energies of 10 KeV and beam currents of 100 mA, the generalized perveance ( $Q$ ) will be  $1.5 \times 10^{-3}$ .

To develop the injection lattice, an appropriate physical model that includes both the influence of space charge as well as the ability to match the beam to the periodical E-Ring structure is required. The KV envelope equations [2] provide a good approximation to the beam dynamics and may be computed quickly enough to support fitting procedures with constraints to obtain the matched machine functions required for the E-ring. However, with the inclusion of bends particularly in the instance of high-current beams with large momentum spread, the KV-model becomes inaccurate. A generalization of the KV-model introduced by A. Garren [3] incorporated a beam dispersion function with the envelope equations into a set of three, second-order differential equations. More recently, M. Venturini and M. Reiser further developed the Garren model by introducing a generalized horizontal emittance  $\epsilon_{dx}$ , combining the second moments and the dispersion [4]. While the standard rms emittance  $\epsilon_x$  due to dispersion is not constant, its generalized version is assumed to be invariant.

Though the model [4] is not fully self-consistent, it is a better approximation to the beam dynamics than the standard KV-model. We have constructed an optimization algorithm using the generalized rms-envelope equations evolving six parameters:  $\sigma_{x,y}$ ,  $\sigma'_{x,y}$  (envelopes and slopes) and  $D_x$ ,  $D'_x$  (horizontal dispersion and its slope) in a set of six first order differential equations [5,6]. The injection system into E-Ring was designed using the KV-envelope equations including dispersion with the design further evaluated by two-dimensional (2D) particle-in-cell (PIC) simulation. In addition, the main E-Ring dynamics were explored by doing turn-by-turn particle tracking using higher-order maps [7].

## 2 Beam matching

The injection system layout, consisting of two dipoles and 8 quadrupole lenses is shown in Fig. 1 where the quadrupoles were assumed to be of the same type as those used for the E-Ring. The first dipole deflects the beam by  $20^\circ$  with a second dipole providing a  $-10^\circ$ . The ring injection is at the dipole magnet labeled D1 is pulsed providing  $-10^\circ$  deflection during deflection and  $+10^\circ$  during beam storage.

The initial electron beam was assumed to have an energy of 10 KeV, a current of 100 mA ( $Q=1.5 \times 10^{-3}$ ), an rms momentum spread  $\sigma_\delta=1.5 \times 10^{-2}$ , and transverse rms-emittances  $\epsilon_{x,y}=12.5\pi$  mm-mrad with rms-envelopes  $\sigma_{x,y}=2$ mm. A fitting procedure was employed to adjust the initial beam parameters to a matched condition in the E-Ring, and the resultant beam RMS envelopes and the dispersion function are shown in Figure 2. The required maximum quadrupole gradient in the injection line is only 10 G/cm, about 20% stronger than the lattice quadrupoles.

Both Figs. 1 and 2 are produced online from the generalized KV-based fitting code. The fitting algorithm required only a few minutes to satisfy the six fitting constraints:  $\sigma_{x,y}$ ,  $\sigma'_{x,y}$ ,  $D_x$ , and  $D'_x$ . Though not shown, the absence of dispersion matching resulted in envelope perturbations in the regular ring structure [5,6]. The research goal for the E-Ring is the understanding of the physics of space charge dominated beams. The avoidance of effects due to mismatched injection would simplify the experimental determination of the parameter relationships. However, for beams with nearly zero momentum spread, the dispersion matching could be ignored and under this circumstance the number of injection line elements could then be reduced.

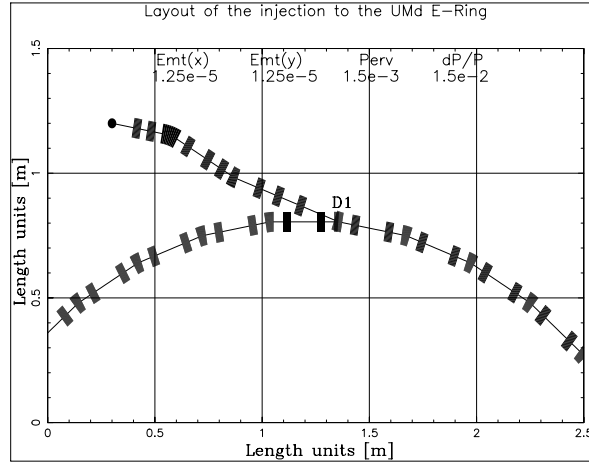


Figure 1: Layout of the injection line into the E-Ring and several sectors of the E-Ring lattice. The blackened circle denotes the input point into the injection system.

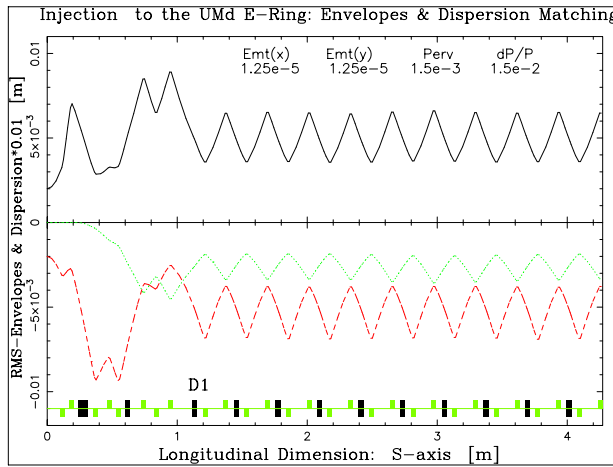


Figure 2: Rms beam envelopes  $\sigma_{x,y}$  (solid and dashed lines) and dispersion function  $D_x$  (dotted line) matched with E-Ring structure. D1 denotes the injection point.

### 3 Two-dimensional PIC studies

A 2D PIC simulation was done to verify the result of Section 2. When the longitudinal forces may be neglected, a 2D PIC formulation provides a self-consistent description of the beam dynamics allowing the inclusion of beam chamber boundaries, non-linear external focusing, and beam momentum spread. Our 2D PIC code had the following features:

- Arbitrary conducting boundaries including free space, rectangular, circular and elliptical.
- Sector dipole, quadrupole, sextupole and octupole magnets.
- Fringing field model for all focusing elements.
- Inclusion of momentum spread.

This code was used to simulate the injection line lattice of Section 2 and further tracking through 36 periods (one full turn of the E-Ring). This analysis had rms-envelopes of the same shape with deviations less than 15% from that shown in Fig. 2 [6].

In the Fig. 3 the behavior of rms-emittances  $\epsilon_{x,y}$  and the generalized version  $\epsilon_{dx}$  is shown. A basic assumption of the generalized rms-envelope equations is that the quantity:  $\epsilon_{dx}^2 = \epsilon_x^2 + \langle \delta^2 \rangle \langle p_x D_x(z) - x D'_x(z) \rangle$  is

constant. However, the results of the PIC simulation show that the generalized emittance  $\epsilon_{dx}$  is not invariant but rather increases from 50 to 70  $\pi$  mm·mrad over a distance of several meters after which it becomes nearly invariant.

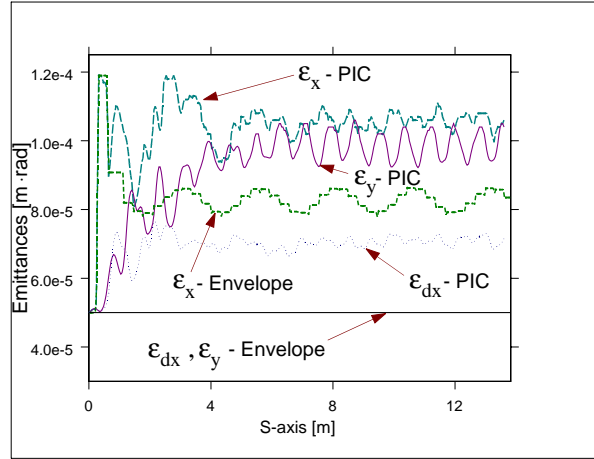


Figure 3: Rms-emittances derived from a generalized KV-envelope model and 2D PIC simulations for the beam injection line and 36 periods (one full turn) of the E-Ring.

#### 4 COSY infinity simulations

To evaluate the E-Ring performance under different space-charge conditions, multi-turn (1000 turn) particle tracking was done [7] with a modified version of the code COSY INFINITY [8] that includes a linear space charge model. In this analysis, the lattice quadrupole settings were maintained at values for a lattice tune of  $\nu_x=7.78$  and  $\nu_y=7.70$  without space-charge. The initial electron beam full (4 x rms) emittance was assumed to 50 $\pi$  mm·mrad (un-normalized). The matched machine functions ( $\beta, \alpha$ ) for the E-Ring lattice for different beam currents were determined and a 5<sup>th</sup> order, one-turn map including the linear space-charge effect was obtained for particle tracking and resonance analysis. The lattice machine functions and tunes of the E-Ring as a function of beam current are given in Tables 1 and 2 under the condition of fixed quadrupole strengths.

Beam Current (mA)	$\nu_x$	$\nu_y$
0	7.780	7.700
1	7.511	7.430
10	5.498	5.411
<b>50</b>	2.040	1.977
100	1.075	1.038

Table 1. E-Ring lattice tune vs. Beam current.

Multi-turn particle tracking was done for four beam current values (1, 10, 50, and 100 mA). The relative emittance dilution or Smear was used to estimate the phase volume dilution and therefore, the beam emittance increase. For purely linear motion, the Smear value is zero. In the presence of non-linear terms and/or mechanical misalignment, mispowering, etc. or space-charge effect, the Smear increases. Figure 4 shows the Smear values vs. the beam current in both transverse planes.

Beam Current (mA)	$\beta_x$	$\alpha_x$	$\beta_y$	$\alpha_y$
0	0.127	0.023	0.505	0.045
1	0.133	0.024	0.519	0.046
10	0.188	0.032	0.682	0.056
<b>50</b>	<b>0.523</b>	<b>0.083</b>	<b>1.797</b>	<b>0.137</b>
100	0.996	0.157	3.412	0.257

Table 2. Matched lattice functions vs. beam current.

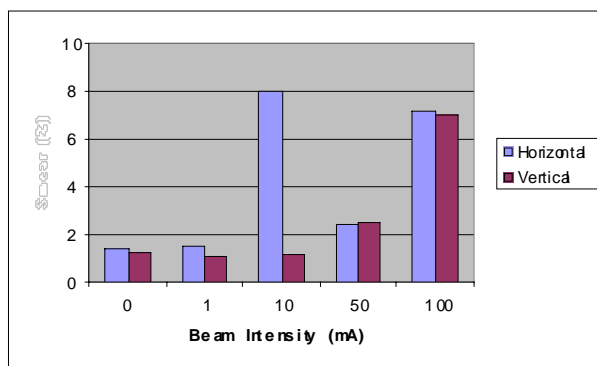


Figure 4: Horizontal and vertical smear values of E-Ring lattice vs. beam current with fixed lattice quadrupole settings.

As shown in Figure 4, without space-charge the Smear in both planes for E-Ring lattice was about 1.5%. The Smear for a beam current of 1 mA was found to be the similar. For a beam current of 10 mA, there is a significant horizontal Smear increase to about 8% while the vertical Smear remains similar to that of non space-charge case. This is due to the horizontal tune for this beam current being  $\nu_x = 5.498$  which is too close to the half-integer resonance of  $2\nu_x = 11$ . The horizontal tune was moved away from the resonance by adjusting the horizontal tune to  $\nu_x = 5.498$ . Under these conditions, the horizontal Smear was reduced to ~ 1.3% while the impact on the vertical Smear was minimal. No similar reduction in Smear was obtained for the other higher currents. See Figure 5.

The larger Smear values for beam currents of 50 and 100 mA are due to the beam-current-driven higher-order geometric aberration terms. The Smear values, however, are still reasonably small (<8%). These results indicate that, although some tune adjustment may be required to avoid resonances, the E-Ring machine operation should be adequate to its maximum design beam intensity.

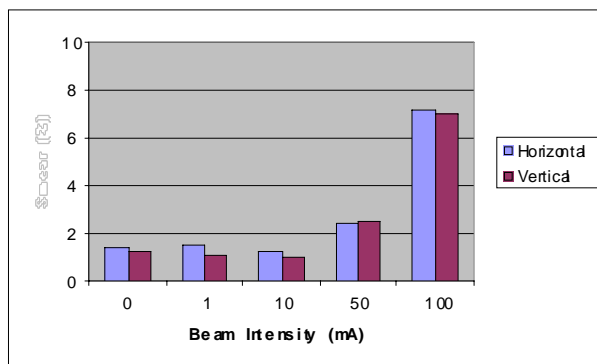


Figure 5: horizontal and vertical smear values of e-ring lattice vs. beam current with adjusted quadrupole settings.

## 5 Conclusions

An envelope-based fitting algorithm was developed for the design of lattices for high-current beams with dispersion. The optimization routines and online visualization of the results make the design process efficient.

A few variants of the E-Ring injection line were found using the fitting algorithm, and the importance of matching a six beam parameters ( $\sigma_{x,y}$ ,  $\sigma'_{x,y}$ ,  $D_x$ , and  $D'_x$ ) for the dynamics in the E-Ring was explored.

The generalized rms-envelope equations were compared to the results of a general 2D PIC code. Turn-by-turn, higher-order simulations, using a linear space charge model, and non-linear terms from, e.g. mechanical misalignment, mispowering etc., showed smear increases below 10% even for the maximum beam current. Tune point adjustment was found to reduce the smear value under certain conditions. Based on these analyses, the E-Ring operation should be adequate.

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