COHERENT AND CHAOTIC PROPERTIES OF NUCLEAR PAIRING

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Pairing interaction, being a very important part of the general residual interaction has specific features being effective for pairs with total spin zero only. The standard approach takes into account the pairing effects with the aid of the BCS approximation borrowed from theory of macroscopic superconductivity. This approximation fails not only from the viewpoint of particle number nonconservation which might be essential for finite systems. It does not work at weak pairing and beyond the phase transition point. Therefore we base our study on the earlier suggested exact numerical solution where the use of the quasispin symmetry provides necessary quantum numbers and extremely simplifies the problem.

The properties of the pairing interaction in the shell model framework are considered with the aid of the exact numerical solution utilizing the quasispin symmetry [1,2],

$$H=\sum_{j}\epsilon_j N_j\,+\,\sum_{j,j'}\mathcal{V}_{jj'}\,P^\dagger_j P_{j'}. \tag{1}$$

Here $P^\dagger_j$ and P_j are pair creation and annihilation operators of particles on a j-level, respectively:

$$P_j=\frac{1}{2}\sum_{m}\tilde{a}_{j\,m}a_{j\,m}\,,\quad P^\dagger_j=(P_j)^\dagger=\frac{1}{2}\sum_{m}a^\dagger_{j\,m}\tilde{a}^\dagger_{j\,m}\,, \tag{2}$$

where the tilde refers to time-conjugate states, $\tilde{a}_{j\,m}\equiv(-)^{j-m}\,a_{j\,-m}$, and $\tilde{\tilde{a}}_{jm}=-a_{jm}$. The operator of the number of particles in a given j-level is $N_j=\sum_m a^\dagger_{jm}a_{jm}$. Three operators,

$$\mathcal{L}^-_j\equiv P_j\,,\quad \mathcal{L}^+_j\equiv P^\dagger_j,\quad \mathcal{L}^z_j\equiv\frac{1}{2}\left(N_j-\frac{\Omega_j}{2}\right), \tag{3}$$

close a quasispin SU(2) algebra; the corresponding classification of the basis is very useful in practical calculations. The quasispin formalism used here reduces the pairing problem to the problem of coupling of partial quasispins $\vec{\mathcal{L}}_j$ for individual j-levels. The diagonalization in the subspace of fixed seniority provides the ground state as well as excited collective states ("pair vibrations").

The usual approximate techniques based on the BCS approach supplemented by the random phase approximation treatment of pair vibrations fail in the region of phase transition, see Fig 1(b). Analogous results can be obtained for isospin-invariant pairing which will be considered elsewhere.

For the first time we have discussed the chaotic (incoherent) aspects of the pairing interaction. The residual interactions in general mix independent particle configurations creating complicated stationary many-body states. These effects influence both local and global statistical properties of the system. In the case of a generic residual interaction, we come locally to the Wigner-like nearest level spacing distribution and enhanced spectral rigidity whereas the global behavior reveals thermalization of the system. Typically, the interaction plays the role of a heat bath bringing the single-particle occupancies close to the Fermi-Dirac distribution even in a strongly interacting system. The pairing interaction is in many respects exceptional. Although it induces the local level repulsion and increases information entropy of the eigenstates, the spectral rigidity shows a pseudo-oscillatory behavior related to the vibrational character of excitations. Both information (Shannon) [3] and invariant (von Neumann) [4] entropies Figs. 1(a) and 1(c) distinctly reflect the existence of the phase transition. The normal behavior of thermodynamic entropy associated with the level density coexists with the absence of full thermalization of single-particle motion.

A pure pairing fermionic condensate has properties similar to those observed in macroscopic superfluid systems. Heating, i.e. the increase of excitation energy, results in gradual destruction of the condensate and

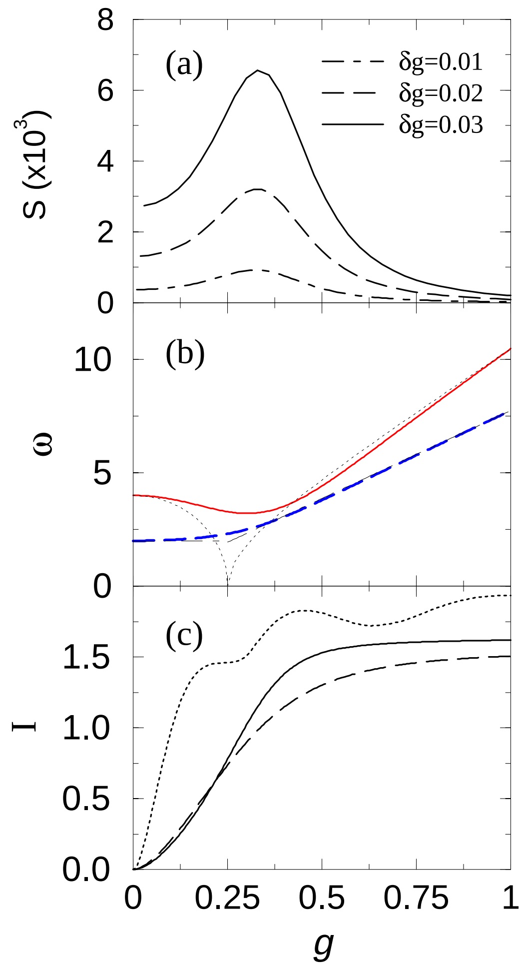

Figure 1: Two-level pairing model; only off-diagonal pair transfer amplitude $\mathcal{V}_{12} = \mathcal{V}_{21} = g$ is taken into account. The upper plot (a) displays the invariant entropy, see text, of the ground state for $\delta g = 0.01,\ 0.02$, and 0.03 averaging intervals. The middle part (b) shows the excitation energy of the lowest pair vibration state, thick solid line. The thin dotted line approximates this curve with the aid of the RPA built on the normal Fermi ground state on the left of the phase transition point at $g = 0.25$ and on the BCS ground state on the right of the critical point. Thick dashed line shows the excitation energy of the lowest state with broken pair, seniority $s = 2$. This curve is compared with thin dash-dot curve, the BCS energy of a two-quasiparticle state. The lower panel (c) displays information entropy for the ground state (solid line), second pair vibration excited state $s = 0$ (dotted line), and the lowest $s = 2$ state (dashed line).

growth of the normal component of the fluid (increase of seniority in the nuclear language). Similarly, a higher angular momentum can be created only by pair breaking. The unpaired particles can be coupled to a nonzero angular momentum which can be treated as some kind of rotation. When the spins allowed for a given seniority are fully aligned, any further increase of angular momentum requires a change of configuration and seniority jump, with the corresponding energy increase. This picture is qualitatively similar to the phenomenon of quantized vortex formation in superfluid liquid ^{4}He.

In Fig. 2 energies of all eigenstates in ^{116}Sn within the shell model space (all possible seniorities) are marked by points in the plane of energy versus angular momentum. The stair-case yrast line, shown by a solid line in Fig 2, is drawn through the states of maximum spin for each seniority. It can be fitted well by a parabola, dashed line, thus giving the average moment of inertia $I = 32$ MeV^{-1} . This result can be compared with the classical moment of inertia of a rigid sphere with uniform density, $I_{\rm rig} = 2MR^2/5$ with mass $M = 938A$ MeV and radius $R = 1.2A^{1/3}$ fm; for ^{116}Sn $I_{\rm rig} = 35$ MeV^{-1} . It is known that in the adiabatic (linear response) approximation a cranked Fermi gas has the rigid body moment of inertia while pairing interactions usually reduce this value [5] by about a factor of 2. The above results show that the overall effect of pairing onto rotational properties is considerably quenched at high excitation energy. The broken pairs and pair vibrations return the moment of inertia to the original Fermi-gas value. This phenomenon has a direct analog in superfluid liquid ^{4}He, where a large number of vortices emerges at high rotation speed forming a lattice structure [6] which rotates as a whole restoring the rigid body moment of inertia and characteristic average velocity field.

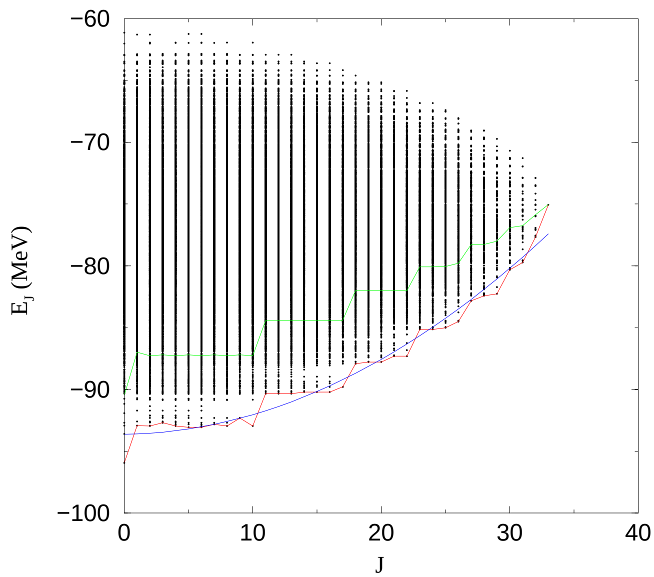

Figure 2: Energies of all many-body states in ^{116}Sn versus their angular momentum; the yrast line (solid line) is fit by a parabola (dashed line); the dashed-dotted curve corresponds to the yrast line in the degenerate case with all single-particle energies being equal.

The pure pairing cannot exist as the only two-body interaction (for a charged system it would violate the gauge invariance, see also [7]). As shown by the full analysis of the statistical properties of the nuclear shell model wave functions [8], the pairing can be considerably modified by the presence of other parts of the residual interaction. For example, the families of states corresponding to various values of the seniority quantum number are almost completely mixed although the pairing phase transition is still observed through the behavior of the pairing correlators for individual low-lying eigenstates. However, beyond the phase transition there exist a long exponentially decreasing tail of dynamic pairing correlations. The interplay of pairing with other types of the residual interaction is an interesting promising problem for future research.

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