RANDOM INTERACTIONS AND COHERENT NUCLEAR STRUCTURE

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The interplay of regular and chaotic elements in quantum many-body dynamics was extensively studied in the framework of random matrix theory and in realistic models of atoms, nuclei, condensed matter and quantum fields. The existence in finite systems of exact conservation laws, such as angular momentum, parity or isospin, raises new questions, for instance, how quantum chaos is influenced by these indestructible symmetries, and what are (if any) the correlations between the blocks of states with different exact quantum numbers governed by the same Hamiltonian. The nuclear shell model with the effective two-body forces in a restricted Hilbert space is the best available theoretical tool for calculating the properties of the low-lying states. Recently [1,2] the low-lying spectra were studied with the shell-model techniques but using, instead of effective interactions, randomly generated (but rotationally-invariant) two-body matrix elements. Some of the results resemble the pattern of actual nuclear spectra. One particularly interesting observation was that of predominance of spin $J = 0$ in the ground state in spite of the low statistical weight of states with $J = 0$ in Hilbert space. This result is robust and insensitive to precise statistical properties of the random interaction. A simple mechanism of random coupling of individual particle spins was suggested in Ref. [3] to explain the preponderance of $J = 0$ (and, in some cases, $J = J_{max}$) in the ground state. In average the yrast-line in a randomly interacting fermionic system acquires a random sign of the effective moment of inertia which leads to the large probabilities of the edge values of the total spin.

Our studies of the wave functions with realistic and random interactions show that the overlap of the 0^+ ground state wave functions generated by random interactions with those obtained for realistic interactions is very small. Also the associated transition probabilities $B(E2)$ to the 2^+_1 state are low. The implication is that the order which is present in actual nuclear states is almost entirely due to the coherent (non-random) aspects of the nuclear Hamiltonian. We need to stress that here we look for the signatures of the coherent phenomena not in the spin ordering which might be a consequence of geometrical constraints, but in the collectivity of the wave functions.

As a generic system we take that of eight particles in the sd-shell, the case corresponding to the ^{24}Mg nucleus. The geometry of the system is much richer than that of the schematic single-j model studied in Ref. [3]; it includes also isospin variables. In the sd model space there are 63 independent matrix elements under constraints of rotational and isospin invariance. We use two interactions, one from Ref. [4] and SDPOTA from Ref. [5], denoted below as (W) and (P), respectively. Being based on different approaches [a fit of individual matrix elements (W) and a fit of a potential (P)], these sets agree in predicting the ground state with quantum numbers $J^{\pi}T = 0^+0$. Our conclusions are essentially the same for both realistic interactions; their ground state wave functions overlap by 98%.

Earlier it was suggested [2] that the wave functions generated by the random interactions carry significant pairing correlations. Having this conjecture in mind, four models of random interactions were considered:

(a) degenerate single-particle energies $0d_{5/2}, 1s_{1/2}, 0d_{3/2}$ (set to zero); all 63 two-body matrix elements generated as random variables uniformly distributed in the interval [-1,1] (in this case the energy scale is arbitrary);
(b) single-particle energies taken from the realistic interaction, W or P, while 63 two-body matrix elements were uniformly generated in the interval $(a - s, a + s)$, where a is the average of the magnitude of the matrix elements in the interaction (W or P), and $s = \sqrt{3}\sigma$; σ is the variance of the matrix elements in the corresponding realistic interaction; the values of a and s are -0.818 MeV and 3.12 MeV, respectively;
(c) the isospin-invariant pairing, $JT = 01$, two-body matrix elements were kept from the realistic interactions whereas the remaining matrix elements were generated as in (b) with the values of a and s equal to -0.616 MeV and 3.03 MeV, respectively;

(d) only six pairing, $JT = 01$, two-body matrix elements were randomly generated in the interval [-1,1] whereas all other matrix elements and all single-particle energies were set to 0; with respect to the energy value, this case can be compared to (a).

		(a)	(b)	(c)	(d)
	% of $JT = 00$ g.s.	59.1	49.3	67.8	92.2
	$\langle E_{00} \rangle$ (MeV)	-13.8	-82.1	-85.0	-3.6
(W)	Average $\langle W \vert R \rangle^2$	0.020	0.053	0.106	0.052
	Variance $\langle W \vert R \rangle^2$	0.056	0.094	0.137	0.096
(P)	Average $\langle P \vert R \rangle^2$	0.019	0.054	0.113	0.061
	Variance $\langle P \vert R \rangle^2$	0.051	0.088	0.137	0.113
	$B(E2)_{av}$	7.0	9.9	14.3	6.2
	$\sigma_{B(E2)}$	10.5	11.9	15.5	5.0
	$B(E2)_{max}$	59.2	59.4	68.0	22.1

Table: Results for models (a), (b), (c), and (d) as described in the text.

The results for 1000 realizations for each model are combined in Table. The ground state energy (relative to ^{16}O and corrected for Coulomb energy as in [4]) with the both realistic interactions is -87.1 MeV. The average ground state energy in cases (b) and (c) is of the same order as in realistic calculations being mainly determined by the single-particle energies. The gain of 2.9 MeV in the version (c) compared to (b) is related to the realistic pairing. The loss of 5 MeV in (b) compared to the full realistic interaction is due to pairing plus multipole-multipole correlations present in the realistic case. The average positions of the ground state in purely random versions (a) and (d) are determined by the widths of the Gaussian many-body level densities known for many-body systems with a random two-body interaction.

For all random ensembles the predominance of the ground states with $J = 0$ is seen clearly. In the fully random case (a) the result is in agreement with what was observed in the pioneering paper [1], confirmed in later studies and attributed mainly to the random geometrical coupling [3]. The presence of regular pairing, case (c), increases the percentage of the ground states with $J = 0$. The strongest effect is observed for the case (d) when the off-diagonal pair transfer matrix elements make quantum numbers $J = T = 0$ preferable for an even number of pairs, as in the case under study, while the competing influence of incoherent interactions is absent.

We studied also the degree of complexity of the eigenstates, the coherence of the wave functions and the quadrupole collectivity. The general conclusion can be formulated as follows. The quadrupole transitions between the lowest 2^+ states and the ground states do not reveal significant collectivity, in contrast with the results of any realistic interaction. In the eigenbasis of the realistic interaction, the states generated by the random interaction are on limit of chaoticity. Small hints of coherent components in the low-lying wave functions generated presumably by the off-diagonal pairing matrix elements and observed also in the earlier studies [3] require a more detail analysis. Both random and realistic interactions can generate regular geometric patterns for the low-lying spectra, but it is only the latter that are relevant for those actually found in nuclear physics.

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