# NUCLEAR REACTIONS -- THEORY

ENERGY-DEPOSITION IN HIGH-ENERGY PROTON INDUCED REACTIONS

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In fragmentation reactions of large targets ($A_T$>50) induced by medium to high energy protons, the inclusive mass-yield of medium mass fragments (e.g. $^{24}$Na has been measured as a function of beam energy.[1] It is always observed, that the production cross-section of these fragments increases strongly with bombarding energy up to a proton kinetic energy of around 5 GeV. For higher incident energies, the fragment yield for these intermediate masses remains constant, indicating that only a finite amount of excitation energy can be deposited in the target.

To investigate this saturation property, we use the previously introduced Boltzmann-Uehling-Uhlenbeck theory[2] describing the time evolution of the phase space distribution function $f(\vec{r},\vec{p},t)$:

$$\frac{\partial f}{\partial t} + \vec{v}\vec{\nabla}_r f - \vec{\nabla}_r U \vec{\nabla}_p f = \frac{1}{(2\pi)^3} \int d^3p_3 d^3p_2 d\Omega v_{12} \frac{d\sigma}{d\Omega}$$

$$\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p})[f_1f_2(1-f_3)(1-f_4)-ff_3(1-f_1)(1-f_2)]$$

To solve this equation the pseudoparticle simulation with 100 parallel runs is used. The mean field potential is parametrized as a function of the density $\rho(\vec{r})$: $U(\rho)=A\rho+B\rho^{\sigma}$. $\sigma$ is a parameter which can be adjusted to obtain the desired compressibility $\kappa$ of nuclear matter. The values of 7/6 and 2 for sigma yield the so-called "soft" and "stiff" equations of state with $\kappa$=200 MeV and 375 MeV, respectively. Here $\sigma$=4/3 was used, yielding $\kappa$=235 MeV.

For the test-simulations the system proton + mass 60 was chosen. The calculations were done in the laboratory frame for central collisions. For the production of medium mass fragments in p-induced reactions only the decay of the spectator-matter can be responsible. Thus the relevant output quantity of our calculations has to be the mean kinetic energy of the spectator nucleons. To discriminate against nucleons being emitted with high energy along the beam directions, we used the prescription

$$p_z \le p_{Max} = \sqrt{2U(\rho_0)m_n} = 320 \text{ MeV/c}$$

Since the present version of the BUU-theory is not able to generate nuclear breakup into composite fragments, the program was turned off after all of the high-energy collisions had stopped. Then the mean kinetic energy was computed and the ground state mean kinetic energy was subtracted. This quantity, $\Delta E_{kin}$, gives a good measure for the excitation energy deposited in the spectators.

The results for $\sigma$=4/3 are displaced in Fig. 1. We have also performed calculations for $\sigma$=2.

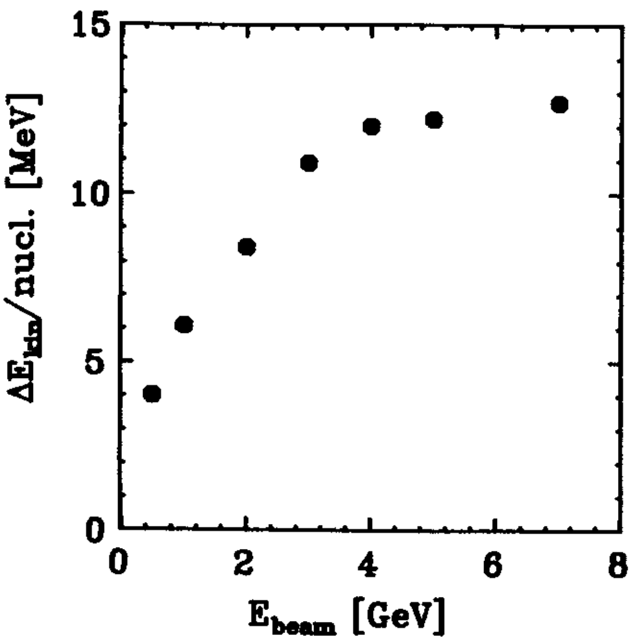


Fig. 1 $\Delta E_{kin}$ of the spectators as a function of beam energy for central collisions of p + mass 60.

The results show practically no deviations from these for $\sigma$=4/3, thus indicating that mean field effects play a minor role and that the process is dominated by collisional dynamics. $\Delta E_{kin}$ increases strongly with beam energy up to a beam energy of 4 GeV. For higher beam energies $\Delta E_{kin}$ does not change any more displaying the saturation property indicated by experiments.

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a. Supported by the Studienstiftung des Deutschen Volkes

References

1. G.D. Cole and N.T. Porile, Phys. Rev. C24,2038(1981).
2. J. Aichelin and G. Bertsch, Phys. Rev. C31,1730(1985).

COMPOSITE FRAGMENT FORMATION IN THE MODIFIED BOLTZMANN-UEHLING-UHLENBECK THEORY

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The Boltzmann equation including the mean field term and the Uehling-Uhlenbeck form of the collision integral provides a numerically tractable theory of the time evolution of the single particle density $f(\vec{r},\vec{p},t)$ in nuclear collisions. However, in its original formulation the theory is completely unrealistic in its predictions of fluctuation phenomena. In particular, the formation of composite fragments cannot be understood in its framework.

We are investigating a modification of the BUU theory treating the collisions as branching points in the density evolution rather than as a continuous source function in the equation of motion. To represent the nucleons we use the usual N test particle simulation with $N \approx 100$ parallel ensembles. But instead of allowing the test particles to collide with cross sections reduced from the physical value by $N^{-1}$ as it is done in the old approach, we allow collisions only with a probability $N^{-1}$ and move 2 N test particles in every allowed collision.

The phase space boundaries for moving these 2 N particles are chosen in the following way: Suppose test particle $i_1$ and $i_2$ in ensemble $\alpha$ have been allowed to collide. Then all other parallel ensembles $\beta$ are searched for the 2 test particles $j_1$ and $j_2$ "closest" to $i_1$ and $i_2$. The "closeness"-conditions are given by:

$$|\vec{r}^{\beta}_{j_1} - \vec{r}^{\alpha}_{i_1}| \le |\vec{r}^{\beta}_{j} - \vec{r}^{\alpha}_{i_1}| \text{ and } |\vec{p}^{\beta}_{j_1} - \vec{p}^{\alpha}_{i_1}| \le \vec{p}_{Fermi}$$

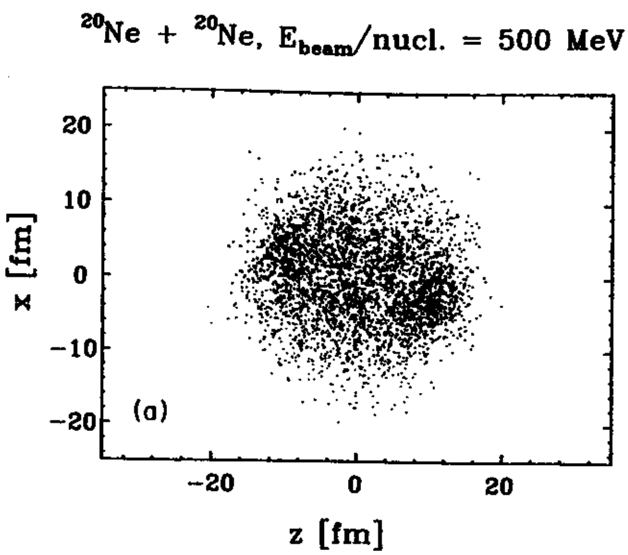
and

$$|\vec{r}^{\beta}_{j_2} - \vec{r}^{\alpha}_{i_2}| \le |\vec{r}^{\beta}_{j} - \vec{r}^{\alpha}_{i_2}| \text{ and } |\vec{p}^{\beta}_{j_2} - \vec{p}^{\alpha}_{i_2}| \le \vec{p}_{Fermi}$$

for all $j \in \beta$.

The two particles $j_1$ and $j_2$ satisfying this condition are then moved into the same phase space region as $i_1$ and $i_2$.

It has to be pointed out that the computational effort is quite large, since one run of the program with N parallel ensembles generates just one event. To generate reasonably high statistics to compare with the experimental mass



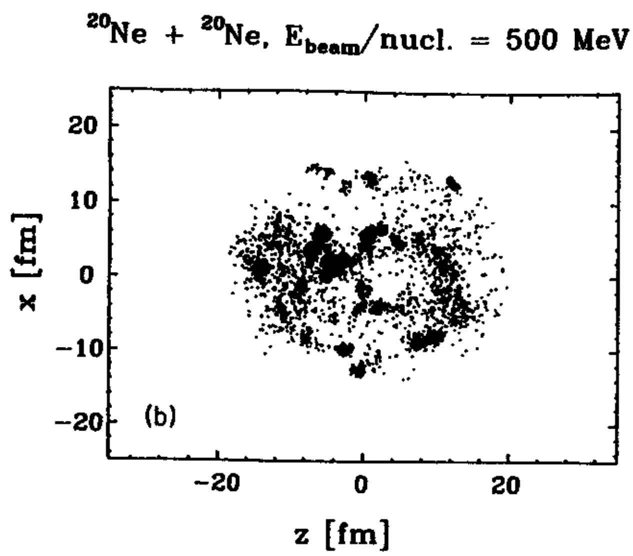




Fig. 1 Coordinate space density distribution $\tilde{\rho}(x,z)$ after 75 time steps of 0.5 fm/c using (a) the old and (b) the new formulation of the theory.

distributions, the power of present day supercomputers is needed.

However, to demonstrate the occurrence of clustering and thus composite fragment production, it is sufficient to perform only a few test runs. In Fig. 1 we show the result of a simulation of a $^{20}Ne + ^{20}Ne$ central collision at 500 MeV beam energy per nucleon after 75 time steps of 0.5 fm/c each. Displayed is the coordinate space density $\bar{\rho}(x,z) = \int \rho(x,y,z)\, dy$. Using the old version of the theory (a), we obtain a rather dilute probabilistic distribution which shows very little clustering. With the new approach (b), the fluctuations created lead to sizable clustering in the coordinate space density which is connected with the production of composite fragments.

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PERCOLATION DESCRIPTION OF NUCLEAR FRAGMENTATION

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In high energy (> 5 GeV) proton induced multifragmentation reactions of heavy targets, a power-law for the absolute cross section of medium mass fragments has been found:[1]

$$\sigma(A_F) \propto A_F^{-\tau};\ \tau \in [2,4]$$

It is possible to understand this power-law in the framework of a percolation description of nuclear fragmentation. We solve the bond percolation problem numerically for a finite three-dimensional simple cubic nucleon lattice.[2] The nucleons are connected to their nearest neighbors via bonds which are broken with a probability p. Using this probability as an input parameter, one can obtain fits to inclusive mass-yield curves and, in particular, reproduce the power-law mentioned above.

In Fig. 1 the results of our model (Nuclear Lattice Model) with p=0.58 are compared to the experimental data of Ref. 1. For the numerical simulation 2500 events have been generated.

To obtain energy distributions of the emitted fragments we use the following prescription: Every nucleon is randomly assigned a charge 0 to 1 with the constraint that the total charge of all nucleons is equal to target plus projectile charge. Furthermore, every nucleon is given a random momentum vector according to a local Thomas-Fermi prescription. To obtain the initial momentum of every fragment, we sum over the momenta of all nucleons in it. here we assume that the linear momentum transfer to the spectators from the collision with the projectile is small compared to the Fermi momentum. Since we also know the intial location of the center of mass of every fragment on the lattice, we can solve the initial value problem for the motion of the fragments under mutual Coulomb repulsion.

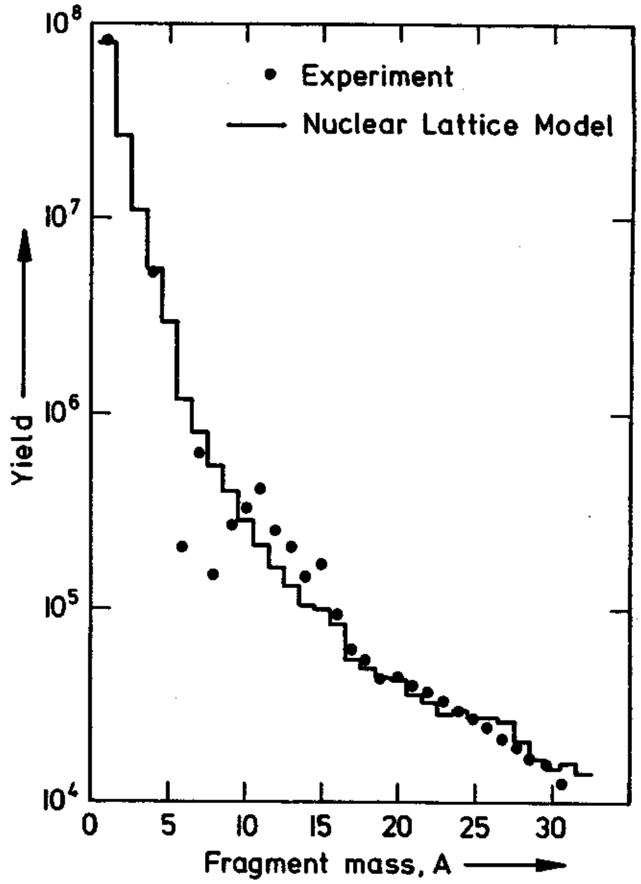


Fig. 1 Comparison of the percolation model fit with p=0.58 to the experimental data from Ref. 1 for the reaction p+Xe at 80-350 GeV.

In Fig. 2 the results of our calculations are compared to the data from Ref. 1. For the numerical simulation 55,000 runs were performed ~500 of these contained a $^{12}C$ fragment.

We are not only able to reproduce the low Coulomb-barrier which is interpreted to be a signature for the simultaneous decay of the spectator matter, but also the slope of the high energy part of the spectrum. Thus we conclude, that the apparent temperatures obtained from medium mass fragment energy distributions can be understood in terms of the Fermi-motion of the nucleons in the target.

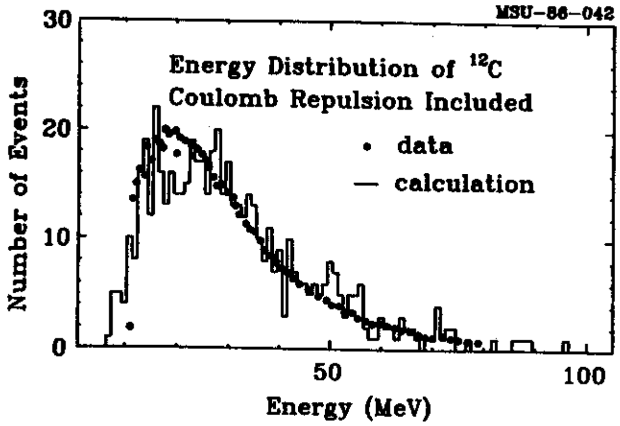


Fig. 2 Energy distribution of $^{12}C$ fragments from the reaction $p+Kr \rightarrow {}^{12}C+X$ at 80-350 GeV. The data are taken from Ref. 1.

a. Supported by the Studienstiftung des Deutschen Volkes

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References

1. A.S. Hirsch et al., Phys. Rev. C29,508 (1984).

2. W. Bauer et al., Phys. Lett. 150B,53(1985).

# HIGH ENERGY PHOTON PRODUCTION IN NUCLEAR COLLISIONS

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Heavy ion collisions produce substantial yields of photons higher in energy than the dipole, which is not well understood theoretically. A basic question is whether the production mechanism is direct or statistical. We have been evaluating two mechanisms for direct production: charge acceleration due to the nuclear potential field and due to first-chance proton-neutron collisions.

These processes are calculated in infinite or semi-infinite nuclear matter, which makes the calculation quite straightforward. Finite size effects are taken into account by assuming that the nucleus can be divided into small noninterfering regions each of which approximates nuclear matter. The neglected interferences give rise to shell effects which are beyond the scope of the theory.

The potential field acceleration mechanism is calculated by taking the proton wave function to be an eigenstate of the one-dimensional Wood-Saxon potential. The resulting photon production rate is averaged over the spherical geometry of the potential edge to account for the finite nuclear geometry. The contribution to the current due to the velocity of the target has also been taken into account. The result for the potential well induced bremsstrahlung is expressed as

$$\frac{d\sigma}{d\omega d\Omega} = 1.2\times10^{-7} \left(\frac{30\mathrm{MeV}}{\omega}\right)^{2.5} \pi R_T^2\left(\frac{2}{3} + \frac{1}{3}\sin^2\theta\right) \left(1-\frac{Z}{A}F(q)\right)^2 \mathrm{MeV}^{-1} \quad (1)$$

where $F(q)$ denotes the target form factor with $F(q=0)=1$, and $R_T$ is the radius of the target nucleus.

For the calculation of bremsstrahlung photons due to the residual interaction in a nucleon-nucleus collision we have considered only collisions between neutrons and protons. Proton-proton bremsstrahlung is smaller by an order of magnitude because the radiation is quadrupolar rather than dipolar. The target nucleus is assumed to be a uniform Fermi sphere in momentum space. The rate is calculated from an integral over two-particle one-hole final states, representing the target nucleus by a Fermi sphere in momentum space. All rates are calculated including effects of Pauli blocking. The bremsstrahlung probability for the first collision is insensitive to the assumed pn interaction. The resulting cross section can be parameterized by the approximate formula

$$\frac{d\sigma}{d\omega d\Omega} = 3\times10^{-7} \frac{(\omega_{max}-\omega)^2}{\omega_{max}\omega} \pi R_T^2 \, g(\theta) \ \mathrm{MeV}^{-1} \quad (2)$$

where $\omega_{max}$ stands for the maximum photon energy and $g(\theta)$ is an angular function normalized to 1 at 90°. It is fairly flat in the mid-velocity frame. A simple comparison of Eqs. (1) and (2) shows that the potential well induced bremsstrahlung is an order of magnitude smaller than that of collisional bremsstrahlung in the intermediate incident energy region. Bremsstrahlung production by proton induced reactions has been measured for 140 MeV proton incident energy on several target nuclei.[1] The collisional process accounts for most of those observed cross sections. For example, the total photon cross section in Pb with $\omega>30$ MeV is $\sigma=250$ μb compared to the observed value of ~225±25 μb.

For nucleus-nucleus collisions, we evaluate the bremsstrahlung rate assuming an intersecting sphere geometry of the momentum distributions.[2] There is no mean field acceleration of the particles to produce this distribution, so the potential well mechanism can be neglected. The bremsstrahlung from first-chance collisions now depends on the pn cross section, unlike the situation in proton-induced reactions. To make a quantitative estimate, we have assumed an interaction strength of magnitude 300 MeV $fm^3$. The nucleon-nucleon cross section is then 40 mb. We also assumed that pp and nn collisions are as likely as pn collisions. The resulting cross section may be parameterized as

$$\frac{d\sigma}{d\omega d\Omega} = 2.2\times10^{-10}\,\frac{(\omega_{max}-\omega)^5}{\omega_{max}^{2.5}\,\omega}\,g(\theta)\pi R_T^2\ \mathrm{MeV}^{-3/2}\ . \quad (3)$$

Here the incident energy dependence (through $\omega_{max}$) is larger than in the proton-nucleus case, because the Pauli principle blocks more phase space. The theory is compared with the recent data by Stevenson et al.)[3] in Fig. 1. The solid line is the prediction from Eq. (3), which falls short of the measured cross section by a factor of 6. This shows that conventional direct mechanisms are not responsible for the observed high energy photons. It is then most likely that the production is a statistical process, but the mechanism needs to be examined more carefully under realistic assumptions about the evolution of the combined system.

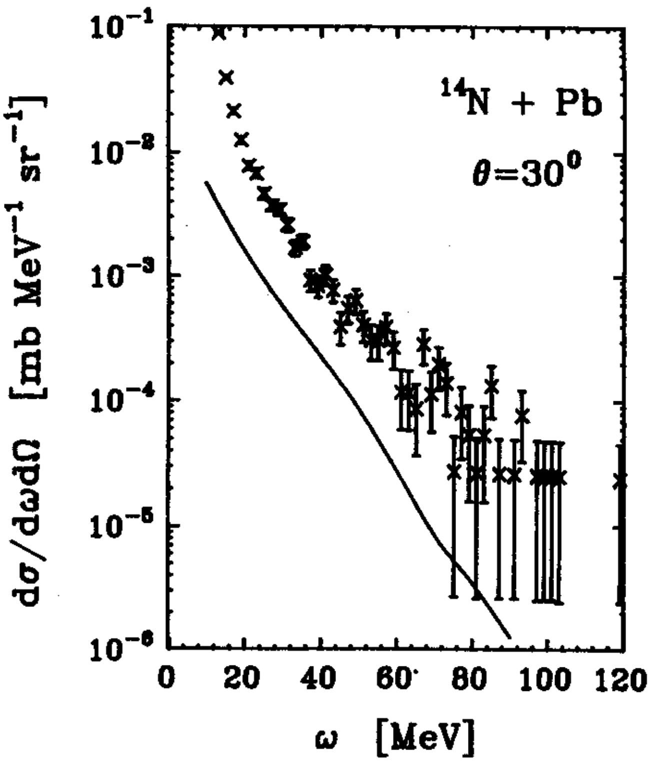


Fig. 1 Energy spectrum of collisional bremsstrahlung photons for the reaction $^{14}$N+Pb at $E_{Lab}$=40 MeV/nucleon and θ=30°. The data are from Ref. 3.

References

1. J.A. Edgington and B. Rose, Nucl. Phys. 89, 523(1966).
2. G.F. Bertsch, Phys. Rev. C15, 713 (1977).
3. J. Stevenson et al., MSU preprint.